Dark energy at cosmological and astrophysical scales: theoretical models and observational tests

B. Novosyadlyj Ivan Franko National University of Lviv, Ukraine

> 3rd Cosmology School Krakow, 10-23 July 2017

The outline of lecture

- 1. Dark energy: definition, discovery, history
- 2. Models of dark energy, dynamical dark energy (DDE)
- 3. Evolution of the DDE in the expanding Universe
- 4. Large scale structure and DDE
- 5. Constraints of parameters of DDE by observational data
- 6. Problems, projects, conclusions

The physical essence which is causing the accelerated expansion of the Universe which is described in the framework of the general relativity (GR):

$$g(r) = -\frac{4\pi}{3}G(\rho + 3p/c^2)r > 0,$$

$$\rho + 3p/c^2 = \rho_m + 3p_m/c^2 + \rho_X + 3p_X/c^2 < 0,$$

$$p_X < -\frac{1}{3}c^2(\rho_m + \rho_X) - p_m$$

Component *X* have been called the dark energy (Huterer D. & Turner M. 1998).

Source of gravitational field: $c^2\rho + 3p = c^2\rho(1+3w)$ Inertial mass: $c^2\rho + p = c^2\rho(1+w)$

Observational evidence for existence of dark energy

- apparent magnitude redshift for SNe Ia,
- apparent magnitude redshift for GRBs,
- acoustic peaks in the angular power spectrum of the CMB,
- baryon acoustic oscillations in the spatial distribution of galaxies,
- angular size redshift for X-ray galaxy clusters,
- formation of the large scale structure of the Universe,
- cross-correlation of ISW effect for CMB and the spatial distribution of galaxies,
- weak gravitational lensing of CMB,
- age of oldest stars in the Galaxy.

"Dark Energy: Mystery of the Millennium" T. Padmanabhan (2006)



- cosmological constant Λ ,
- scalar field (quintessence, phantom, quintom, K-essence, tachyon field, Chaplygin gas, barotropic fluid ...) which almost homogeneously fills the Universe,
- more general theory than GR or another gravitation theory (Brans-Dicke theory, f(R)-gravity, dilaton gravity, MOND...).

Century of cosmological constant

Albert Einstein in 1917 has added the cosmological constant into equations of General Relativity in order to obtain the model of eternal static world



142 Sitzung der physikalisch-mathematischen Klasse vom 8. Februar 1917

Kosmologische Betrachtungen zur allgemeinen Relativitätstheorie.

Von A. Einstein.

Es ist wohlbekannt, daß die Poissonsche Differentialgleichung

$$\Delta \phi = 4\pi K \rho \tag{1}$$

in Verbindung mit der Bewegungsgleichung des materiellen Punktes die Newtonsche Fernwirkungstheorie noch nicht vollständig ersetzt. Es muß noch die Bedingung hinzutreten, daß im räumlich Unendlichen das Potential ϕ einem festen Grenzwerte zustrebt. Analog ver144 Sitzung der physikalisch-mathematischen Klasse vom 8, Februar 1917

der an sich nicht beansprucht, ernst genommen zu werden: er dient nur dazu, das Folgende besser hervortreten zu lassen. An die Stelle der Poissonschen Gleichung setzen wir

$$\Delta \phi - \lambda \phi = 4\pi h^{2}, \qquad (2)$$

wobei λ eine universelle Konstante bedeutet. Ist ρ_0 die (gleichmäßige) Dichte einer Massenverteilung, so ist

$$\phi = -\frac{4\pi K}{\lambda} z_0 \tag{3}$$

eine Lösung der Gleichung (z). Diese Lösung entspräche dem Falle, daß die Materie der Fixsterne gleichmäßig über den Raum verteilt wäre, wobei die Dichte z_0 gleich der tatsächlichen mittleren Dichte der Materie des Weltraumes sein möge. Die Lösung entspricht einer unendlichen Ausdehnung des im Mittel gleichmäßig mit Materie erfüllten Raumes. Denkt man sich, ohne an der mittleren Verteilungsdichte etwas zu ändern, die Materie örtlich ungleichmäßig verteilt, so wird sich über den konstauten ϕ -Wert der Gleichung (z) ein zuEinstein: Kosmologische Betrachtungen zur allgemeinen Relativitätstheorie 151

müßten wir wohl schließen, daß die Relativitätstheorie die Hypothese von einer räumlichen Geschlossenheit der Welt nicht zulasse.

Das Gleichungssystem (14) erlaubt jedoch eine naheliegende, mit dem Relativitätspostulat vereinbare Erweiterung, welche der durch Gleichung (2) gegebenen Erweiterung der Poissonschen Gleichung vollkommen analog ist. Wir können nämlich auf der linken Seite der Feldgleichung (13) den mit einer vorläufig unbekannten universellen Konstante $-\lambda$ multiplizierten Fundamentaltensor g_{ur} hinzufügen, ohne daß dadurch die allgemeine Kovarianz zerstört wird; wir setzen an die Stelle der Feldgleichung (13)

$$G_{\mu\nu} - \lambda g_{\mu\nu} = - \varkappa \left(T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} T \right). \qquad (13a)$$

Auch diese Feldgleichung ist bei genügend kleinem λ mit den am Sonnensystem erlangten Erfahrungstatsachen jedenfalls vereinbar. Sie befriedigt auch Erhaltungssätze des Impulses und der Energie, denn man gelangt zu (13a) an Stelle von (13), wenn man statt des Skalars des Riemansschen Tensors diesen Skalar, vermehrt um eine universelle

"Kosmologische Betrachtungen zur allgemeinen Relativitatsdtheorie"

152 Sitzung der physikalisch-mathematischen Klasse vom 8. Februar 1917

$$-\frac{2}{R^2} + \lambda = -\frac{\kappa\rho}{2}$$
$$-\lambda = -\frac{\kappa z}{2}$$
$$\lambda = \frac{\kappa z}{2} = \frac{1}{12}$$
(14)

oder

Die neu eingeführte universelle Konstante λ bestimmt also sowohl die mittlere Verteilungsdichte z, welche im Gleichgewichte verharren kann, als auch den Radius R des sphärischen Raumes und dessen Volumen $2\pi^*R^3$. Die Gesamtmasse M der Welt ist nach unserer Auffassung endlich, und zwar gleich

 $-R^{2}$

$$M = z \cdot 2 \pi^2 R^3 = 4 \pi^2 \frac{R}{z} = \frac{\sqrt{32\pi^2}}{\sqrt{2^3z}}.$$
 (15)

Die theoretische Auffassung der tatsächlichen Welt wäre also, falls dieselbe unserer Betrachtung entspricht, die folgende. Der Krümmungscharakter des Raumes ist nach Maßgabe der Verteilung der Materie zeitlich und örtlich variabel, läßt sich aber im großen durch einen

Discovery of the accelerated expansion of the Universe (1998)

SuperNova Cosmology Project

High-z SuperNova Search

$$d_L \equiv \sqrt{\frac{L}{4\pi F}} = \frac{c(1+z)}{H_0} \int_0^z \frac{dz'}{\sqrt{\Omega_m (1+z')^3 + \Omega_\Lambda}}$$
$$(m-M) = 5 \log d_L + 25 + \alpha(s-1) - \beta \mathcal{C},$$
$$(z \equiv \Delta \lambda / \lambda = 1/a(t) - 1)$$

Discovery of the accelerated expansion of the Universe (1998)

From the measurements $d_L(z)$ of light curves of 50 SNe Ia carried out by SNCP and HzSNS teams

 $q_0 = -0.54 \pm 0.2$ ($\ddot{a} > 0$) at 3σ ($\approx 99.7\%$) C.L.!

Perlmutter et al. (1999):

 $\Omega_m - 0.75\Omega_{\Lambda} = -0.25 \pm 0.125 \quad \rightarrow \quad \Omega_{\Lambda} = 0.71 \pm 0.07$

Riess et al. (2004): $\Omega_{\Lambda} = 0.71^{+0.03}_{-0.05}$ (1 σ C.L.)

Key experiments of 1998-2011 years

- 1998 HzSNST (SN Ia, Riess et al.)
- 1998 SNCP (SN Ia, Perlmutter et al.)
- 1999 Toco (CMB, *Miller et al.*)
- 2000 Boomerang (CMB, *Bernardis et al.*)
- 2000 MAXIMA (CMB, *Hanany et al.*)
- 2001 DASI (CMB, Halverson et al.)
- 2002 ACBAR (CMB, Kuo et al.)
- 2003 WMAP-1 (CMB, Spergel et al.)
- 2005 BAO (SDSS, Eisenstein et al.)
- 2006 WMAP-3 (CMB, Spergel et al.)
- 2008 WMAP-5 (CMB, Komatsu et al.)
- 2011 WMAP-7 (CMB, Komatsu et al.)
- 2011 ACT (CMB, *Dunkley et al.*)

"For the discovery of the accelerating expansion of the Universe through observations of distant supernovae".

Perlmutter S. (team SNCP), Nature, **391**, 51 (1998) Perlmutter S. (team SNCP), ApJ. **517**, 565 (1999)

Riess A. G. (HzSNS), AJ. 116, 1009 (1998)

Schmidt B. P. (HzSNS), ApJ. 507, 46 (1998)

Problems of Λ

- Is Λ the second gravitational constant? If "yes", then what means that value of $\Lambda = 3H_0^2\Omega_{\Lambda} \propto 10^{-56} \,\mathrm{cm}^{-2}$ (or $\propto 10^{-122}$ against G = 1 in the Planck units)?
- Is Λ a measure of vacuum energy (Zeldovich, 1968)?

If "yes", then why $\rho_{\Lambda} = 3H_0^2 \Omega_{\Lambda}/8\pi G \propto 10^{-29} \text{ g/cm}^3$ is 10^{-54} orders of magnitude smaller than the modern prediction considering the vacuum energy of all known scalar and vector fields (Martin, 2012)?

- Fine-tunning problem: at the end of inflations (reheating) $\rho_{\Lambda}/\rho_{m+\gamma} \sim 10^{-96}$. Why? Current physics does not explain...
- Coincidence problem: at the current epoch $\rho_{\Lambda} \approx \rho_m$, at the epoch of reionization $\rho_{\Lambda} \approx \rho_{\gamma}$ (arXiv:1707.03388). Why? Current physics does not explain...

Anthropic principle (Weinberg, 1987) as solution: if the cosmological constant were only one order of magnitude larger than its observed value, the universe would suffer catastrophic inflation, which would preclude the formation of stars, and hence life.

Dynamical DE $\delta \rho_{de} \neq 0, \quad V_{de} \neq 0$ quintessence phantom quintom **K**-essence scalar fields tachyon fields Chaplygin gas

Non-dynamical DE $\delta \rho_{de} = 0, \quad V_{de} = 0$ Λ -model vacuum fields f(R)-gravity MOND holographic dark energy

non-minimally coupled

Scalar field as dark energy

$$\mathcal{L}(X, U(\phi)), \qquad X \equiv \frac{1}{2}\phi_{;i}\phi^{;i}$$

$$T_{ij} = \mathcal{L}_{,X}\phi_{,i}\phi_{,j} - g_{ij}\mathcal{L}$$

$$T_{ij} = (\rho_{de} + p_{de})u_iu_j - p_{de}g_{ij}$$

$$p_{de} = \mathcal{L} \qquad \qquad \rho_{de} = 2X\mathcal{L}_{,X} - \mathcal{L}$$

$$w_{de} \equiv \frac{p_{de}}{\rho_{de}} = \frac{\mathcal{L}}{2X\mathcal{L}_{,X} - \mathcal{L}} < -\frac{1}{3} \qquad c_s^2 \equiv \frac{\delta p_{de}}{\delta \rho_{de}} = \frac{\mathcal{L}_{,X}}{\mathcal{L}_{,X} + 2X\mathcal{L}_{,XX}} \ge 0$$
$$\Omega_{de} \equiv \frac{\rho_{de}}{\rho_{cr}} = \frac{8\pi G}{3H_0^2}\rho_{de} \qquad c_a^2 \equiv \frac{\dot{p}_{de}}{\dot{\rho}_{de}}$$

Scalar field as dark energy: a few examples of Lagrangians

Lagrangian	EoS	Effect. sound speed	References
$\mathcal{L} = \frac{1}{2}\phi_{;i}\phi^{;i} - V(\phi)$	$w_{de} = \frac{\dot{\phi}^2 - 2V(\phi)}{\dot{\phi}^2 + 2V(\phi)}$	$c_{s}^{2} = 1$	[1-6]
$\mathcal{L} = -\frac{1}{2}\phi_{;i}\phi^{;i} - V(\phi)$	$w_{de} = \frac{-\dot{\phi}^2 - 2V(\phi)}{-\dot{\phi}^2 + 2V(\phi)}$	$c_{s}^{2} = 1$	[7-13]
$\mathcal{L} = -V(\phi)\sqrt{1 - \phi_{;i}\phi^{;i}}$	$w_{de} = \dot{\phi}^2 - 1$	$c_s^2 = -w_{de}$	[14-21]
$\mathcal{L} = -V(\phi)\sqrt{1+\phi_{;i}\phi^{;i}}$	$w_{de} = -\dot{\phi}^2 - 1$	$c_s^2 = -w_{de}$	[22]
$\mathcal{L} = F(X) - V(\phi)$	$w_{de} = \frac{F - V}{2XF' - F + V}$	$c_s^2 = \frac{F'}{F' + 2XF''}$	[23]
$\mathcal{L}\left(\phi, X, U(\phi) ight)$	$w_{de} = rac{\mathcal{L}}{2X\mathcal{L}_{,X} - \mathcal{L}}$	$c_s^2 = rac{\mathcal{L}_{,X}}{\mathcal{L}_{,X} + 2X\mathcal{L}_{,XX}}$	[24-30]

[1] Ratra & Pebbles (1988); [2] Wetterich (1988); [3] Peebles & Ratra (1988); [4] Turner & White (1997); [5] Caldwell et al. (1998); [6] Zlatev et al. (1999);
[7] Caldwell (2002); [8] Caldwell et al. (2003); [9] Fabris & Concalves (2006); [10] Kujat et al. (2006); [11] Lima & Pereira (2008); [12] Schrerrer & Sen (2008);
[13] Creminalli et al. (2009); [14] Padmanabhan (2002); [15] Gibbons (2002); [16] Frolov et al. (2002); [17] Bagla et al. (2003); [18] Abramo & Finelli (2003);
[19] Gorini et al. (2004); [20] Sen (2005); [21] Calcagni & Liddle (2006); [22] Babichev et al. (2006, 2008); [23] Haq Ansari & Unnikrishnan (2011);
[24] Armendariz-Picon et al. (2001); [25] Malquarti et al. (2003); [26] Malquarti et al. (2003); [27] de Putter & Linder (2007); [28] Aguirregabiria et al. (2005);
[29] Bilic (2008); [30] Bilic et al. (2009).

Complete references are in the book at arXiv:1502.04177

Potential	Where did it come from; References		
$V = M^{4-n} \phi^n, n > 0$	particle physics; Linde (1990)		
$V = M^{4+n} \phi^{-n}, n > 0$	SUSY; Binetruy (1998); Masiero et al. (1999); SG; Brax & Martin (1999); Copeland et al. (2000);		
$V = \Sigma_n a_n \phi^n$	polynomial potential		
$V = M^4 \exp\left(-\beta \phi/M_p\right)$	moduli; Ferreira & Joyce (1998); dilaton field; Barreiro et al. (2000);		
$V = M^4 \exp\left(M_p/\phi\right)$	exponential tracker fied		
$V = M^{4+n} \phi^{-n} \exp\left(\alpha \phi^2 / M_p^2\right)$	SUSY; Binetruy (1998); Masiero et al. (1999); SG; Brax & Martin (1999); Copeland et al. (2000);		
$V = M^4 \cos^2\left(\phi/2f\right)$	pseudo-Nambu-Goldstone boson; Frieman et al. (1995)		

EoS parametrization			
$w_{de} = const$	1-parametric		
$w_{de} = w_0 + w_a \frac{z}{z+1}$	2-parametric CPL; Chevallier & Polarski (2001); Linder (2003)		
$w_{de} = w_0 + w_a \frac{z}{(z+1)^2}$	2-parametric; Bagla et al. (2003);		
$w_{de} = w_0 + w_a \frac{z(z+1)}{z^2 + 1}$	2-parametric; Barboza & Alcaniz (2008);		
$w_{de} = \frac{w_0}{w_0 + (1 + w_0)(z + 1)^{3(1 + w_a)}}$	2-parametric GCG; Thakur et al. (2012);		
$w_{de} = w_0 + w_a \left(\frac{z}{z+1}\right)^7$	2-parametric; Pantazis et al. (2016);		
$w_{de} = \frac{z_{tr}+1}{z+z_{tr}+2} \left[w_0 + w_a \frac{z}{z+1}\right] - \frac{z+1}{z+z_{tr}+2}$	3-parametric; Komatsu et al. (2009);		
$w_{de} = w_0 + \frac{(w_f - w_0)}{1 + \exp(\frac{z - z_{tr}}{\Delta})}$	4-parametric; Bassett et al. (2002);		

Scalar field as dark energy: freezing and thawing

$$T_{0;i}^{i\,(de)} = 0:$$

$$\frac{dw_{de}}{d\ln a} = 3(1+w_{de})(w_{de} - c_a^2)$$

Classes of quintessence scalar fields, freezing ($w'_{de} < 0$) and thawing ($w'_{de} > 0$) ones, in the phase plane $w_{de}(a) - w'_{de}(a)$. Black solid lines show the boundaries of these classes in the phase space, the short-dashed line shows the boundary between field evolution accelerating and decelerating down the potential. The brown and blue lines show evolutionary tracks of scalar fields with above potentials. The arrows show the direction of evolution from beginning (a = 0) to current epoch (a = 1).

(From Caldwell R.R. and Linder E.V., Phys. Rev. Lett. 95, 141301 (2005)).

I. Scalar field as dynamical dark energy at cosmological scales

Dark energy and expansion of the Universe

Einstein equations :
$$R_{ij} - \frac{1}{2}g_{ij}R = \kappa \left(T_{ij}^{(r)} + T_{ij}^{(m)} + T_{ij}^{(de)}\right)$$

Friedmann metric : $ds^2 = g_{ij}dx^i dx^j = c^2 dt^2 - a^2(t)\delta_{\alpha\beta}dx^{\alpha}dx^{\beta}$
EoS equation : $p = wc^2\rho$ $(w_r = \frac{1}{3}, w_m = 0, w_{de} < -\frac{1}{3})$
 $T_{i;k}^{k(r)} = 0 \rightarrow \dot{\rho}_r = -4\frac{\dot{a}}{a}\rho_r \rightarrow \rho_r(t) = \rho_r^0 a^{-4}$
 $T_{i;k}^{k(m)} = 0 \rightarrow \dot{\rho}_m = -3\frac{\dot{a}}{a}\rho_m \rightarrow \rho_m(t) = \rho_m^0 a^{-3}$
 $T_{i;k}^{k(de)} = 0 \rightarrow \dot{\rho}_{de} = -3(1+w_{de})\frac{\dot{a}}{a}\rho_{de} \rightarrow \rho_{de}(t) = \rho_{de}^0 a^{-3(1+\tilde{w}_{de})},$

where

$$\tilde{w}_{de}(a) = \frac{1}{\ln(a)} \int_{1}^{a} w_{de}(a') d\ln a' \quad \text{and} \quad \tilde{w}_{de} = w_{de} \quad \text{for} \quad w_{de} = const$$

Dark energy and expansion of the Universe

Figure 1: The evolution of energy density of relativistic (r), matter (m) and dark energy w = const (de) components. RDE - Radiation Dominated Epoch, MDE -Matter Dominated Epoch and DEDE - Dark Energy Dominated Epoch. All lines correspond to model with $\Omega_m = 0.3$ and $\Omega_{de} = 0.7$. The MDE-DEDE crossing line is shown for Λ -model ($w_{de} = -1$).

Dark energy and expansion of the Universe

$$H \equiv \frac{\dot{a}}{a}, \qquad q \equiv -\frac{\ddot{a}}{aH^2}$$

$$H = H_0 \sqrt{\Omega_r a^{-4} + \Omega_m a^{-3} + \Omega_k a^{-2} + \Omega_{de} f(a)},$$
$$q = \frac{1}{2} \frac{2\Omega_r a^{-4} + \Omega_m a^{-3} + (1 + 3w)\Omega_{de} f(a)}{\Omega_r a^{-4} + \Omega_m a^{-3} + \Omega_k a^{-2} + \Omega_{de} f(a)},$$

where

$$\Omega_x \equiv \frac{\rho_x^{(0)}}{\rho_{cr}^{(0)}}, \quad \rho_{cr}^{(0)} \equiv \frac{3H_0^2}{8\pi G}, \quad f(a) \equiv \frac{\rho_{de}(a)}{\rho_{de}^0} = a^{-3(1+\tilde{w}_{de}(a))}$$

Why does dynamical dark energy evolve?

• because space-time evolves:

$$R = 6\left(\frac{\ddot{a}}{a} - \frac{\dot{a}^2}{a^2}\right) = -6\left(1 + q\right)H^2$$

• inherent property

Specifying of scalar field models of dark energy

$$\frac{p_{de}}{\dot{\rho}_{de}} = c_a^2 = const \quad \rightarrow \quad p_{de} = c_a^2 \rho_{de} + C$$

[Babichev, Dokuchaev & Eroshenko (2005)]

$$\frac{dw_{de}}{da} = 3a^{-1}(1+w_{de})(w_{de}-c_a^2)$$

$$w_{de}(a) = \frac{(1+c_a^2)(1+w_0)}{1+w_0 - (w_0 - c_a^2)a^{3(1+c_a^2)}} - 1$$

$$\rho_{de}(a) = \rho_{de}^{(0)} \frac{(1+w_0)a^{-3(1+c_a^2)} + c_a^2 - w_0}{1+c_a^2}$$

$$f(a) = \frac{(1+w_0)a^{-3(1+c_a^2)} + c_a^2 - w_0}{1+c_a^2}$$

Phase plane and evolution of EoS parameter of DE

Left phase plane $dw_{de}/d\ln a - w_{de}$ for dark energy models with $-2 \le c_a^2 \le 0$. Colors denote the regions occupied by models with $-1.1 \le w_{de} \le -0.9$ at the current epoch: blue – models with decreasing w_{de} and raising repulsion (freezing quintessence dark energy), green – models with increasing w_{de} and receding repulsion (thawing quintessence dark energy, which becomes "false phantom" one with $w_{de} < -1$ but $\dot{p}_{de} < 0$, $p_{de} < 0$, $p_{de} > 0$ in far future for short time in the vicinity of turnaround point, red – phantom models with decreasing w_{de} and raising repulsion of dark energy. Right: examples of evolution tracks of EoS parameter w_{de} from the colored ranges on the left. The dotted line represents the subclass of dark energy models with $w_0 < -1$ and $c_a^2 > w_0$, for which $\rho_{de} < 0$ at some time in the past and which is excluded from further analysis.

Dynamics of expansion of the Universe with scalar field dark energy

Future of the Universe depends on the nature of dark energy

$$t(a) = \int_0^a \frac{da'}{a'H(a')} \quad \to \quad a(t)$$

Big Rip singularity: $t_{BR} - t_0 \approx \frac{2}{3} \frac{1}{H_0} \frac{1}{|1+c_a^2|} \sqrt{\frac{1+c_a^2}{(1+w_0)\Omega_{de}}}$

Reconstruction of Lagrangian of scalar field

For
$$c_s^2 = const$$
 we obtain $L = VX^{\frac{1+c_s^2}{2c_s^2}} - U$

$$U = \frac{\rho_{de}(c_s^2 - w_{de})}{1 + c_s^2},$$

$$V = V_0(c_s^2 - w_{de})\rho_{de},$$

$$X = \left(\frac{c_s^2}{1 + c_s^2} \frac{|1 + w_{de}|}{c_s^2 - w_{de}}\right)^{\frac{2c_s^2}{1 + c_s^2}} (\pm V_0)^{-\frac{2c_s^2}{1 + c_s^2}}$$

[Sergijenko & Novosyadlyj, Phys.Rev.D, 92 (2015); arXiv:1407.2230]

Potential and kinetic term for different values of c_s^2

$$w_{de} = -0.9$$

$$w_{de} = -1.1$$

Perturbations:

 δ

$$T_{ij} = \overline{T}_{ij} + \delta T_{ij}$$

$$\rho = \overline{\rho}(1+\delta), \quad p = \overline{p}(1+\pi), \quad u^{i} = \overline{u}^{i} + \delta u^{i},$$

$$\phi = \overline{\phi} + \delta\phi,$$

$$\phi_{de} = \left(\dot{\phi}\dot{\delta}\dot{\phi} - \Psi\dot{\phi}^{2}\right) \left(\frac{\partial\mathcal{L}}{\partial X} + 2X\frac{\partial^{2}\mathcal{L}}{\partial X^{2}}\right) - \left(\frac{\partial\mathcal{L}}{\partial U}\frac{\partial U}{\partial \phi} - 2X\frac{\partial^{2}\mathcal{L}}{\partial X\partial U}\frac{dU}{d\phi}\right)\delta\phi,,$$

$$\delta p_{de} = \left(\dot{\phi}\dot{\delta}\dot{\phi} - \Psi\dot{\phi}^{2}\right)\frac{\partial\mathcal{L}}{\partial X} + \frac{\partial\mathcal{L}}{\partial U}\frac{\partial U}{\partial \phi}\delta\phi,$$

$$V_{de} = \frac{k\delta\phi}{\dot{\phi}}$$

 $ds^{2} = c^{2}dt^{2} + a^{2}(t)(\delta_{ij} + h_{ij})dx^{i}dx^{j} \quad (h \equiv h_{i}^{i} \ll 1),$ $R_{ij} = \overline{R}_{ij} + \delta R_{ij}, \quad R = \overline{R} + \delta R$

Formation of large scale structure: some basic equations

$$\delta R_j^i - \frac{1}{2} \delta_j^i \delta R = \kappa \left(\delta T_j^{i(r)} + \delta T_j^{i(m)} + \delta T_j^{i(de)} \right)$$

Equations for Fourier modes of perturbations in the synchronous gauge comoving to matter component ($V_m = 0$):

$$\dot{\delta}_{de} + 3(c_s^2 - w_{de})aH\delta_{de} + (1 + w_{de})\frac{\dot{h}}{2} + (1 + w_{de})\left[k + 9a^2H^2\frac{c_s^2 - c_a^2}{k}\right]V_{de} = 0,$$

$$\dot{V}_{de} + aH(1 - 3c_s^2)V_{de} - \frac{c_s^2k}{1 + w_{de}}\delta_{de} = 0,$$

$$\dot{\delta}_m = -\frac{1}{2}\dot{h}_s$$

$$\ddot{h} + \frac{a}{a}\dot{h} = -8\pi Ga^2(\rho_m\delta_m + (1+3w_{de})\rho_{de}\delta_{de})$$

CAMB: http://camb.info.Lewis A., Challinor A, Lasenby A., Astrophys. J. 538, 473 (2000)

Evolution of matter density perturbations in the models with different types of DE

The evolution of matter density perturbations from the Dark Ages to the present epoch in sCDM, Λ CDM, QSF+CDM and PSF+CDM models (amplitudes are normalized to 0.1 at a = 0.1):

sCDM:
$$\Omega_m = 1$$
;
 Λ CDM: $\Omega_m = 0.3$, $\Omega_{de} = 0.7$;
QSP1: $\Omega_m = 0.3$, $\Omega_{de} = 0.7$, $w_0 = -0.8$, $c_a^2 = -0.8$
QSP2: $\Omega_m = 0.3$, $\Omega_{de} = 0.7$, $w_0 = -0.8$, $c_a^2 = -0.5$
PSP1: $\Omega_m = 0.3$, $\Omega_{de} = 0.7$, $w_0 = -1.2$, $c_a^2 = -1.2$;
PSP2: $\Omega_m = 0.3$, $\Omega_{de} = 0.7$, $w_0 = -1.2$, $c_a^2 = -1.5$;

Evolution of density perturbations in the models with phantom dark energy

The evolution of different Fourier amplitudes of PSF and matter density perturbations from a = 0.1 to a = 200 for models with $w_0 = -1.2$, $c_a^2 = -1.5$ and $w_0 = -1.2$, $c_a^2 = -1.2$.

The different lines correspond to different wave numbers k (in Mpc⁻¹) as follows: 1 - 0.0005, 2 - 0.001, 3 - 0.0015, 4 - 0.002, 5 - 0.0025, 6 - 0.005, 7 - 0.01, 8 -0.05, 9 - 0.1 Mpc⁻¹.

[Novosyadlyj, Sergijenko, Durrer & Pelykh, PhysRev D 86 (2012)]

Density perturbations of dark matter, baryon matter and dark energy ($k=0.1 Mpc^{-1}$) (CAMB)

[Sergijenko & Novosyadlyj, Phys.Rev.D, 92 (2015)]

Influence of c_s^2 on CMB power spectrum of $\Delta T/T$ (CAMB)

$$w_{de} = -0.9, c_a^2 = -0.5 \qquad \qquad w_{de} = -1.1, c_a^2 = -1.5$$

Observational data and method of determination of cosmological parameters

Observational data

CMB: Planck

CMB: WMAP9

BAO: SDSS DR7

BAO: 6dF

BAO: SDSS DR9

SNe la: SNLS3

SNe Ia: Union2.1

Planck collaboration (2015) Hinshaw et al. (2013) Percival et al. (2010) Beutler F. et al. (2011) Anderson et al. (2012) Sullivan et al. (2011) Suzuki et al. (2012)

Method

Markov Chain Monte Carlo CosmoMC [Lewis & Bridle (2002)]

Theoretical predictions vs observational data

CMB:

$$\frac{\ell(\ell+1)}{2\pi}C_{\ell}^{TT} = \langle (\Delta T)^2 \rangle_{\theta \approx \pi/\ell} , \quad \frac{(\ell+1)}{2\pi}C_{\ell}^{TE} = \langle \Delta T \cdot E \rangle_{\theta \approx \pi/\ell}$$

Power spectrum of matter density perturbation:

$$P(k) \equiv \langle \delta(k)\delta^*(k) \rangle = A_s k^{n_s} T^2(k;\Omega_i, w_0, c_a^2)$$

Its amplitude:

$$\sigma_8^2 = \frac{1}{2\pi^2} \int_0^\infty k^2 P(k) W^2(8Mpc \cdot k/h) dk, \quad W(x) = 3\frac{\sin x - x\cos x}{x^3}$$

BAO:

$$R(z) \equiv \frac{r_s(z_{drag})}{D_V(z)}$$

SNe la:

$$(m - M) = 5 \log d_L + 25 + \alpha(s - 1) - \beta C,$$
$$d_L = \frac{c(1 + z)}{H_0} \int_0^z \frac{dz'}{\sqrt{\Omega_m (1 + z')^3 + \Omega_{de} f(\frac{1}{1 + z'})}}.$$

Likelihood function:

$$L(\mathbf{x};\theta_k) = exp\left(-\frac{1}{2}(x_i - x_i^{th})C_{ij}(x_j - x_j^{th})\right) \approx exp\left(-\frac{1}{2}\sum_i \frac{(x_i - x_i^{th})^2}{\sigma_i^2}\right)$$

Posterior function:

$$P(\theta_k; \mathbf{x}) = \frac{L(\mathbf{x}; \theta_k) p(\theta_k)}{g(\mathbf{x})}$$

Parameters:

$$\theta_k$$
: $\Omega_{de}, w_{de}, c_a^2, c_s^2, \Omega_b, \Omega_{cdm}, H_0, A_s, n_s, \tau_{rei}$

Anisotropy of cosmic microwave background (key experiments)

WMAP

Planck

South polar telescopes SPT BICEP2

ACT

Planck $\Delta T/T$ -map after data processing and map cleaning (98.4% of sky)

Planck results 2015: TT, TE and EE power spectra

Planck Collaboration, arXiv:1502.01582

Comparison with other measurements of D_l

Planck results: cosmological parameters of ΛCDM **model**

Parameter	Planck TT+lowP+lensing		
$\overline{\Omega_{\rm b}h^2}$	0.02226 ± 0.00023		
$\Omega_c h^2$	0.1186 ± 0.0020		
$100\theta_{MC}$	1.04103 ± 0.00046		
τ	0.066 ± 0.016		
$\ln(10^{10}A_s)$	3.062 ± 0.029		
<i>n</i> _s	0.9677 ± 0.0060		
H_0	67.8 ± 0.9		
$\Omega_{\rm m}$	0.308 ± 0.012		
$\Omega_{\rm m}h^2$	0.1415 ± 0.0019		
$\Omega_{\rm m}h^3$	0.09591 ± 0.00045		
σ_8	0.815 ± 0.009		
$\sigma_8 \Omega_m^{0.5} \dots \dots$	0.4521 ± 0.0088		
Age/Gyr	13.799 ± 0.038		
<i>r</i> _{drag}	147.60 ± 0.43		
$k_{\rm eq}$	0.01027 ± 0.00014		

Planck Collaboration, arXiv:1502.01582

The best-fit values and 2σ confidence limits for parameters of cosmological models

Table 1: The best-fit values (p_i) , mean values and 2σ confidence limits for parameters of cosmological models using 4 different observational datasets: WMAP9 + HST + BAO + SNLS3 (p_1) , WMAP9 + HST + BAO + SN Union2.1 (p_2) , Planck + HST + BAO + SNLS3 (p_3) , Planck + HST + BAO + SN Union2.1 (p_4) .

Parameters	\mathbf{p}_1	2σ c.l.	\mathbf{p}_2	2σ c.l.	\mathbf{p}_3	2σ c.l.	\mathbf{p}_4	2σ c.l.
Ω_{de}	0.727	$0.722^{+0.022}_{-0.023}$	0.720	$0.718 \substack{+0.023 \\ -0.025}$	0.718	$0.719^{+0.021}_{-0.023}$	0.721	$0.717 \substack{+0.023 \\ -0.024}$
w_0	-1.123	$-1.120 \substack{+0.160 \\ -0.156}$	-1.114	$-1.092 \substack{+0.181 \\ -0.190}$	-1.146	$-1.169^{+0.139}_{-0.136}$	-1.247	$-1.158 \substack{+0.165 \\ -0.156}$
c_a^2	-1.171	$-1.337^{+0.322}_{-0.288}$	-1.341	$-1.282 \substack{+0.731 \\ -0.342}$	-1.152	$-1.372^{+0.235}_{-0.242}$	-1.599	$-1.374 \substack{+0.246 \\ -0.238}$
$10\Omega_b h^2$	0.225	$0.225 \substack{+0.009 \\ -0.009}$	0.226	$0.225 \substack{+0.009 \\ -0.009}$	0.220	$0.221 \substack{+0.005 \\ -0.005}$	0.220	$0.221 \substack{+0.005 \\ -0.005}$
$\Omega_{cdm}h^2$	0.118	$0.117 \substack{+0.006 \\ -0.006}$	0.118	$0.117 \substack{+0.006 \\ -0.006}$	0.121	$0.120 \substack{+0.004 \\ -0.004}$	0.121	$0.120 \substack{+0.004 \\ -0.004}$
h	0.718	$0.711 \substack{+0.028 \\ -0.029}$	0.709	$0.704 \substack{+0.032 \\ -0.031}$	0.713	$0.714 \substack{+0.027 \\ -0.027}$	0.718	$0.710^{+0.030}_{-0.030}$
n_{S}	0.968	$0.968 \substack{+0.022 \\ -0.021}$	0.970	$0.969^{+0.023}_{-0.022}$	0.958	$0.960 {+0.012 \atop -0.012}$	0.960	$0.960^{+0.012}_{-0.012}$
$\log(10^{10}A_s)$	3.103	$3.096 \substack{+0.059 \\ -0.056}$	3.095	$3.097 \substack{+0.059 \\ -0.055}$	3.098	$3.089^{+0.050}_{-0.047}$	3.090	$3.088 \substack{+0.050 \\ -0.047}$
$ au_{rei}$	0.087	$0.086 \substack{+0.027 \\ -0.026}$	0.082	$0.087 \substack{+0.026 \\ -0.026}$	0.093	$0.089 \substack{+0.026 \\ -0.024}$	0.089	$0.089^{+0.026}_{-0.024}$

[Novosyadlyj B., Sergijenko O., Durrer R., Pelykh V., JCAP, 5, 30 (2014)]

CMB: theoretical predictions vs observational data

SNe Ia: theoretical predictions vs observational data

Arbitrating power of observational data on CMB anisotropy

Left panel: CMB angular power spectra for the cosmological models with best-fit parameters $\mathbf{p}_1 - \mathbf{p}_4$ (superimposed lines) are compared to currently available data (symbols). Right panel: the relative differences of temperature and polarization power spectra for models with different \mathbf{p}_i compared with experimental uncertainties.

Relative differences of the theoretical predictions from the model with parameters p_i versus the relative uncertainties of observational data for BAO's (left panel) and SNe Ia distance moduli (right panel).

Observational data: Planck, BAO, SNIa, WL, RSD, H₀

 $\Omega_{\Lambda} = 0.692 \pm 0.012$

[Planck Collaboration, A&A, 594, id.A14 (2016)]

Current determination of dark energy parameters

Observational data: cosmic shear, galaxy-galaxy lensing, galaxy clustering

[Dark Energy Survey Collaboration, arXiv:1706.09359 (2017)]

Current determination of dark energy parameters

Observational data: Planck, WiggleZ, SN Union2.1, H₀

[Sergijenko & Novosyadlyj, Phys.Rev.D, 92 (2015)]

- Observational data prefer the cosmological model with DE density domination at current epoch: $\Omega_{de} = 0.7 \pm 0.02$. The model without DE ($\Omega_{de} = 0$) is excluded at > 50 σ C.L. !
- Observational data related with cosmological scales (Planck results 2015) give strong constraints on the density of dark energy in the early Universe: $\Omega_{EDE} < 0.0071$.
- Observational data related with cosmological scales do not distinguish the DE type: $w_0 = -1.0 \pm 0.15$.
- Currently available observational data related with cosmological scales give no possibility to constrain c_s^2 !

Current and future projects

- The Dark Energy Survey (DES) is an international, collaborative effort to map hundreds of millions of galaxies, detect thousands of supernovae, and find patterns of cosmic structure that will reveal the nature of the mysterious dark energy that is accelerating the expansion of our Universe. DES began searching the Southern skies on August 31, 2013.
- Euclid (ESA) is a mission to map the geometry of the dark Universe: will investigate the distance-redshift relationship and the evolution of cosmic structures by measuring shapes and redshifts of galaxies and clusters of galaxies out to redshifts 2.

Current and future projects

- The goal of the Large Synoptic Survey Telescope (LSST) project is to conduct a 10-year survey of the sky that will deliver a 200 petabyte set of images and data products that will address some of the most pressing questions about the structure and evolution of the universe and the objects in it.
- Wide-Field Infrared Survey Telescope (WFIRST, NASA) is mission to perform an extraordinarily broad set of scientific investigations: studying the newly-discovered phenomenon of dark energy, measuring the history of cosmic acceleration, completing the exoplanet census begun by NASA's Kepler Space Telescope and demonstrating technology for direct imaging and characterization of exoplanets.

Special issue on dark energy, Eds. G. Ellis, H. Nicolai, R. Durrer, R. Maartens, Gen. Relat. Gravit. 40 (2008)

Amendola L. and Tsujikawa S., *Dark Energy: Theory and Observations*, Cambridge University Press, 507 p. (2010)

Lectures on Cosmology: Accelerated expansion of the Universe. Lect. Notes in Physics **800**, Ed. G. Wolschin, Springer, 188 p. (2010)

Dark Energy: Observational and Theoretical Approaches, Ed. P. Ruiz-Lapuente, Cambridge University Press, 321 p. (2010)

Novosyadlyj B., Pelykh V., Shtanov Yu., Zhuk A., *Dark energy: observational evidence and theoretical models*, ed. V. Shulga, Akademperiodyka, Ukraine, 381 p. (2013)

Cosmology and Fundamental Physics with the Euclid Satellite, Amendola et al. (The Euclid Theory Working Group) (2016), arXiv:1606.00180 Thank you for attention!

Much ado about nothing?

Marginal evidence for cosmic acceleration from Type Ia supernovae

J. T. Nielsen¹, A. Guffanti², and S. Sarkar^{1,3}

¹Niels Bohr International Academy, Blegdamsvej 17, Copenhagen 2100, Denmark
 ² Universit degli Studi di Torino, via P. Giuria 1, I-10125 Torino, Italy and
 ³Rudolf Peierls Centre for Theoretical Physics, 1 Keble Road, Oxford OX1 3NP, UK

The 'standard' model of cosmology is founded on the basis that the expansion rate of the universe is accelerating at present — as was inferred originally from the Hubble diagram of Type Ia supernovae. There exists now a much bigger database of supernovae so we can perform rigorous statistical tests to check whether these 'standardisable candles' indeed indicate cosmic acceleration. Taking account of the empirical procedure by which corrections are made to their absolute magnitudes to allow for the varying shape of the light curve and extinction by dust, we find, rather surprisingly, that the data are still quite consistent with a constant rate of expansion.

I. INTRODUCTION

In the late 1990's, studies of Type Ia supernovae (SN Ia) showed that the expansion rate of the universe appears to be accelerating as if dominated by a cosmological constant is Since then supernova cosmology has developed rapidly as an important probe of 'dark enconstants are fitted along with the cosmological parameters. The physical mechanism(s) which give rise to the correlations that underlie these corrections remain uncertain^{20/21]}. The SN Ia distance modulus is then compared to the expectation in the standard ACDM cosmological model:

IS THE EXPANSION OF THE UNIVERSE ACCELERATING? ALL SIGNS POINT TO YES

D. RUBIN^{1,2} AND B. HAYDEN^{2,3} Draft version December 20, 2016

ABSTRACT

The accelerating expansion of the universe is one of the most profound discoveries in modern cosmology, pointing to a universe in which 70% of the mass-energy density has an unknown form spread uniformly across the universe. This result has been well established using a combination of cosmological probes (e.g., Planck Collaboration et al. 2016), resulting in a "standard model" of modern cosmology that is a combination of a cosmological constant with cold dark matter and baryons. The first compelling evidence for the acceleration came in the late 1990's, when two independent teams studying type Ia supernovae discovered that distant SNe Ia were dimmer than expected. The combined analysis of modern cosmology experiments, including SNe Ia, the Hubble constant, baryon acoustic oscillations, and the cosmic microwave background has now measured the contributions of matter and the cosmological constant to the energy density of the universe to better than 0.01, providing a secure measurement of acceleration. A recent study (Trøst Nielsen et al. 2015) has claimed that the evidence for acceleration from SNe Ia is "marginal." Here we demonstrate errors in that analysis which reduce the acceleration significance from SNe Ia, and further demonstrate that conservative constraints on the curvature or matter density of the universe increase the significance even more. Analyzing the Joint Light-curve Analysis supernova sample, we find 4.2σ evidence for acceleration with SNe Ia alone. and 11.2σ in a flat universe. With our improved supernova analysis and by not rejecting all other cosmological constraints, we find that acceleration is quite secure.

Subject headings: cosmology: observations, cosmology: cosmological parameters, cosmology: dark energy