Conceptual issues in cosmology: inflationary paradigm and horizons

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Cosmological puzzle

Advent of physical cosmology in 1965: the U. with its variety of constituent objects realizes the simplest evolving cosmological model (solution) in GR. It is:
— highly homogeneous,
— almost prefectly isotropic,
— spatially flat,
— very old and very large (the observable U.).
Why is it so? The current ΛCDM model is exceptional in the class of Robertson–Walker spacetimes and R–W spacetimes are a very special case of possible cosmological worlds (are very little probable).
Common opinion for decades:
the U. should be a generic cosmol. solution of EFE.
The opposite occurs and possible explanations of the simplicity of the U.:
1) Anthropic Principle (Brandon Carter 1973), weak and strong — the U. is ,,biophilic",
2) inflationary evolution of the very early U.

Cosmological horizons

That the U. is bizarre is best illustrated in terms of cosmological horizons. They are important for cosmology (most textbooks are careless on them). Robertson–Walker spacetime:

$$ds^2 = c^2\,dt^2 - R^2(t)[dr^2 + f_k^2(r)(d\theta^2 + \sin^2\theta\,d\phi^2)],$$

$$f_k(r) = \begin{cases} r & \text{for } k = 0 \\ \sin r & \text{for } k = +1 \\ \sinh r & \text{for } k = -1. \end{cases}$$

r — dimensionless radial coordinate, $[R] =$ length.
EFE hold $\Rightarrow$ spacetime singularities exist. R–W geometry: initial singularity (Big Bang) — singular (all distances $= 0$) spacelike hypersurface $t = 0 \Leftrightarrow R(0) = 0$.
All cosmic particles = galaxies =fundamental observers at rest =comoving reference frame,
$x^0 = ct,\ r,\ \theta,\ \phi$ — comoving coordinates,
$r, \theta, \phi =$ const — timelike geodesics.

I. Particle horizon

All cosmol. horizons depend on the observer's worldline and the observation point on it.
E — Earth with $r = 0$, its geodesic worldline begins at singularity at $t = 0$.
Particle horizon: limits the part of the U. with which we can have causal connection at t_0.

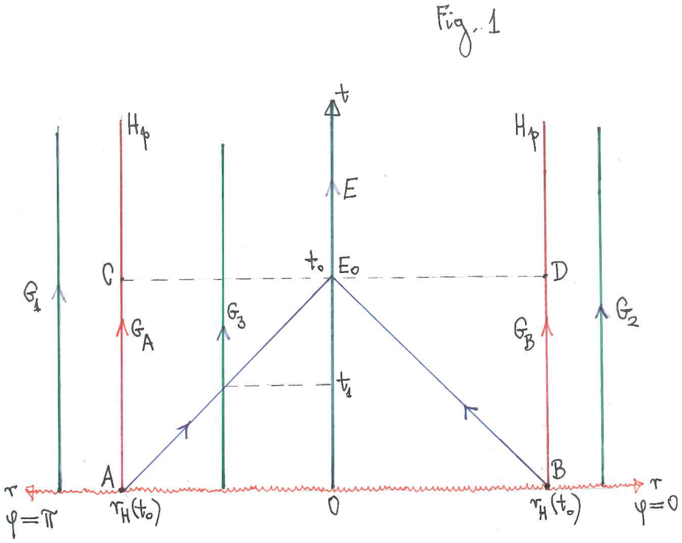
Fig. 1
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$\varphi=\pi$
$r_H(t_0)$
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$r_H(t_0)$
$\varphi=0$

Fig. 1

G_A, G_B, G_1, G_2, G_3 — geodesic worldlines of galaxies.
Observer at $E_0(t_0)$ sees G_A and G_B emerging from the singularity at antipodal points A, B — the farthest objects seen at t_0.
G_1, G_2 — not seen yet at t_0,
G_3 — seen as emitting at t_1.
Particle horizon H_p of E at t_0:
3–dim. timelike hypersurface (tube=cylinder) comprised of worldlines of G_A, G_B seen at t_0 as emerging from BB.
A, B — coordinates $r_A = r_B \equiv r_H(t_0), \theta, \phi$ — form a sphere of intersection of the past light cone of $E_0(t_0)$ with the singul.
Observer E_0 can see only events on and inside the past light cone AE_0B.

The causal limit sphere at t_0.
CD — sphere of points on H_p simultaneous with $E_0(t_0)$,
$r_H(t_0)$ — radial coordinate of all points on $H_p(E, t_0)$.
Radial null geodesic from A (or B) to E_0:

$$ds^2 = c^2\, dt^2 - R^2\, dr^2 = 0, \quad c\, dt = -R\, dr, \quad c\frac{dt}{R} = -dr| \int_{r_H}^{0},$$

$$r_H(t_0) = c \int_0^{t_0} \frac{dt}{R(t)},$$

converges for Friedmann models.

Distances in cosmology.

Proper distance from $E_0(t_0, r = 0, \theta, \phi)$ to $C(t_0, r_H(t_0), \theta, \phi)$ — length of spacelike radial curve with $t = t_0$ and $\theta, \phi =$const,

$$d_p(E_0, C) \equiv R(t_0) r_H(t_0),$$

NOT the geodesic distance and non–measurable (frequent error!).

Radius of $H_p(t_0)$: *proper distance* from E_0 to points C, D of the causal limit sphere — ,,radius" of this sphere,

$$d_H(t_0) \equiv R(t_0)\, r_H(t_0) = c\, R(t_0) \int_0^{t_0} \frac{dt}{R(t)}.$$

d_H is non–measurable as concerns points simultaneous with E_0, yet it depends only on time.

Example.

Let $k = 0$, $p = w\rho$, $w =$const $\Rightarrow$

$$R = \mathrm{const} \cdot t^{\frac{2}{3(1+w)}}, \qquad d_H(t) = \frac{3(1+w)}{1+3w}\, ct.$$

For $t_0 \approx 13,7 \cdot 10^9$ years and $w = 0$: $d_H(t_0) \approx 4 \cdot 10^{10} \mathrm{ly} \approx 10\,\mathrm{Gpc}.$

H_p expands at light velocity $\Leftrightarrow r_H(t)$ grows in time as $r(t)$ along a radial null geodesic. Proof.

$$\frac{d}{dt}\, r_H(t) = \frac{c}{R(t)}.$$

A photon is emitted radially outwards at $r(t_0) < r_H(t_0)$. Along its path $ds^2 = 0 \Rightarrow c\,dt = R\,dr$ and

$$\frac{dr}{dt} = \frac{c}{R(t)} = \frac{dr_H}{dt},$$

or $r(t) < r_H(t)$ forever. A photon initially inside H_p will never reach $r_H(t) \Rightarrow$ it will not escape outside and must remain inside.
Hence: a massive particle once inside H_p cannot leave it.

Warning.

In the case of the inflation this property of H_p is frequently distorted and presented in an erroneous way.

Wikipedia: Inflation (cosmology).

,,A space with Λ is qualitatively different: instead of moving outward, the cosmological horizon stays put. For any one observer, the distance to the cosmological horizon is constant. With exponentially expanding space, two nearby observers are separated very quickly; so much so, that the distance between them quickly exceeds the limits of communications. The spatial slices are expanding very fast to cover huge volumes. Things are constantly moving beyond the cosmological horizon, which is a fixed distance away, and everything becomes homogeneous.

As the inflationary field slowly relaxes to the vacuum, the cosmological constant goes to zero and space begins to expand normally. The new regions that come into view during the normal expansion phase are exactly the same regions that were pushed out of the horizon during inflation, and so they are at nearly the same temperature and curvature, because they come from the same originally small patch of space."

Yet $d_H(t)$ suggests that H_p expands faster than light: its radial velocity is

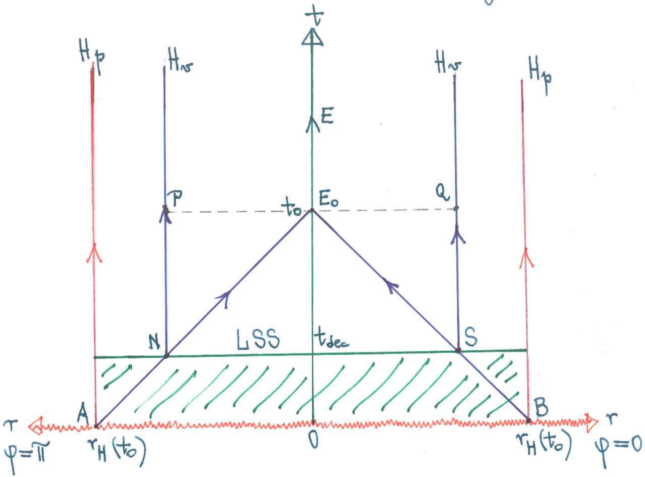
$$v_r \equiv \frac{d}{dt} d_H(t) = c[1 + \frac{\dot{R}}{R} d_H(t)] = \frac{3(1+w)}{1+3w}$$

dust: $w = 0 \Rightarrow v_r = 3c$,
radiation: $w = 1/3 \Rightarrow v_r = 2c$.
This is NOT a physical motion. H_p expands as a light cone and $d_H(t)$ is quantitatively misleading.
Misleading terminology:
,,particles (galaxies) enter our H_p" — incorrect,
,,our H_p expands and includes new comoving (or not) particles" — correct.

II. Visual horizon

H_p is the limit of what in principle (by any physical means) may be observed (detected) by us at t_0. If a particle worldline lies on $H_p(t_0)$ we see it at $t = 0 \Rightarrow$ its light signal has $z = \infty$ — it is physically unobservable. Moreover: very early U. is a hot dense charged plasma $\Rightarrow$ the U. is opaque. We actually see signals emitted at $t \geq t_{\rm dec}$ — after the recombination.
Visual horizon (optical horizon) H_v — *border of the domain about which we can have any observational evidence (by elm. signals)* = 3–dim. timelike hypersurface (cylinder) comprising worldlines of (comoving) particles that emitted the CMB radiation at $t_{\rm dec}$ and is observed now $\Rightarrow H_v$ is inside H_p.
H_v is determined by intersection of past light cone of $E_0(t_0)$ with the last scattering space at $t = t_{\rm dec}$ — it is the sphere comprising points N and S („relic sky")= the last scattering surface observed by WMAP and Planck. Fig. 2

Fig. 2



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$r_H(t_0)$
0
$r_H(t_0)$
$\varphi=0$

The radial coordinate of H_v is determined by null radial geodesic from N (or S) to E_0,

$$r_{vh} = c \int_{t_{\rm dec}}^{t_0} \frac{dt}{R} < r_H(t_0),$$

is independent of the evolution for $t < t_{\rm rec}$.
At $t = t_0$ H_v forms a sphere of points P, Q — **visual limit sphere** with the radius (proper distance)

$$d_{vh}(t_0) = R(t_0)\, r_{vh}(t_0),$$

it contains all matter around us that has been and is observed by elm. signals.

Consequences of the existence of H_p and H_v.
They represent absolute limits on what is observable and testable in the U.
They determine causal limitations in structure formation in the early U.
Current speculations about global („super–horizon") structure of the U.
— chaotic inflation — are not observationally testable.
The observable U. is inside H_v — „Metagalaxy". All physical theories are tested within Metagalaxy.
In principle we can get signals from quadrangle ANSB: neutrinos and primordial gravit. waves — far future.
The U. itself is much bigger than Metagalaxy. GR: matter outside H_p affects gravitationally matter in Metagalaxy — but we cannot decode it to learn about distribution and motion of galaxies outside H_p
I emphasize: all inquiries about the U. as a whole („multiverse") are unverifiable.

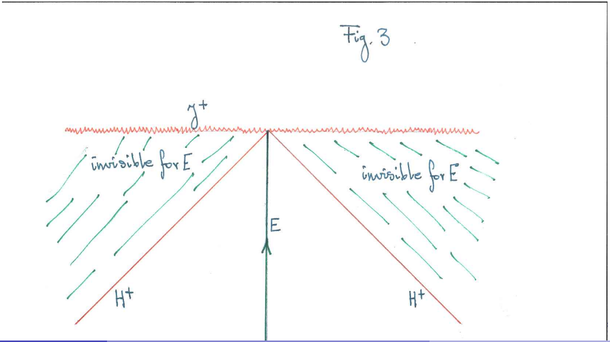
III. Cosmological event horizons

Also in geometries other than R–W. There are past and future cosmol. event horizons, more interesting are future ones.

$\mathcal{J}^+$ — spacelike future (final) singularity. Worldline of E (not necessarily a timelike geodesic) ends on $\mathcal{J}^+$.

Cosmol. future event horizon $H^+(E)$ — *past light cone of the final point of E* (intersection with $\mathcal{J}^+$.

$H^+(E)$ separates spacetime events that will be ever observable by E from those that will not.



Simplest example:
radiation dominated ($p = \frac{1}{3}\rho$) closed ($k = +1$) Friedmann universe. There is initial and final singularity. Here $0 \leq r \leq \pi$ and E: $r = 0$. Parametrized solution:

$$\begin{aligned} R &= R_M \sin\eta, \\ ct &= R_M(1 - \cos\eta), \quad 0 \leq \eta \leq \pi \quad \text{— ,,conformal'' time}, \\ R(0) &= 0 \quad \text{—Big Bang}, \quad R(\frac{\pi}{2}) = R_M \quad \text{— maximal size}, \\ R(\pi) &= 0 \quad \text{— big crunch}, \\ t(0) &= 0, \quad t(\frac{\pi}{2}) = \frac{R_M}{c}, \quad t(\pi) = \frac{2R_M}{c} \equiv t_f \quad \text{—collapse}. \end{aligned}$$

$H_p(t_f)$ has

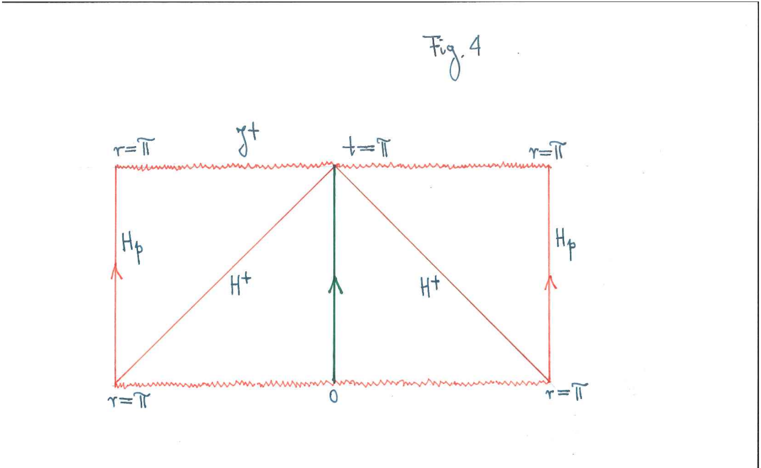
$$r_H(t_f) = c\int_0^{t_f} \frac{dt}{R} = c\int_0^{\pi} \frac{1}{R}\frac{dt}{d\eta}\, d\eta = \pi.$$

$H^+(E)$ intersects initial singularity surface at $r = r_e$, then $ds^2 = 0 \Rightarrow$

$$r_e = c\int_0^{t_f} \frac{dt}{R} = \pi = r_H(t_f).$$

Here $H_p(t_f)$ contains the whole space S^3 and approaching the big crunch observer E will see all objects in the world, the farthest just emerging from Big Bang.

Event horizons in cosmology are close to absolute event horizons in black holes. In cosmology are of little significance — they refer to far future or to big crunch.



Hubble sphere

Improperly called ,,Hubble horizon".
The proper distance from us to a galaxy at r is $d_p = R(t)r$.
Lemaître–Hubble law for ,,escape velocity" is

$$v \equiv \frac{d}{dt} d_p = \dot{R}r = H(t)\, d_p, \quad H \equiv \frac{\dot{R}}{R} \quad \text{— Hubble function.}$$

Erroneous claim: for $d_p = c/H$ the escape velocity is c and farther galaxies are invisible and cannot communicate with us.
The Hubble sphere with proper radius c/H is important in the theory of density perturbations in the early U., it is *not* a horizon.

Problems of the decelerating early U.

Existence of cosmol. horizons is a property of many models (solutions) within R–W spacetime. They are *not* a problem for these models. They reveal deep problems that cosmology has with R–W geometry.
These problems arise from the observation that to look like it does today, the U. would have to have started from *very finely tuned* initial conditions soon after Big Bang.
Key assumption (belief) in physics:
realistic models of the U. should not be sensitive to initial conditions.
This is assumption — there is no physical principle requiring this. Comes from experience in laboratory physics: what is probable and what is not.
The standard cosmol. model rests on highly special initial conditions:
— R–W spacetime since $t > t_P \approx 10^{-43}$ s,
— almost (or exact) flat space,
— domination of ultrarelat. matter („radiation") from $\approx t_P$ to $t_{\rm rec} \approx 10^6$ years.
For a long time these have seemed *unnatural*.

I emphasize: only quite recently it has been recognized that there is no physical principle excluding the possibility that the U. had very special initial conditions.

This exclusion was based on application of the notion of probability to the *unique* U.

Common conviction was:

the U. emerged from the initial singul. as a generic cosmol. solution: inhomogeneous and anisotropic. Soon after it a kind of physical process started at some place in space to operate and smooth out the geometry and matter distribution.

Physical nature of this process — unknown and has never been investigated.

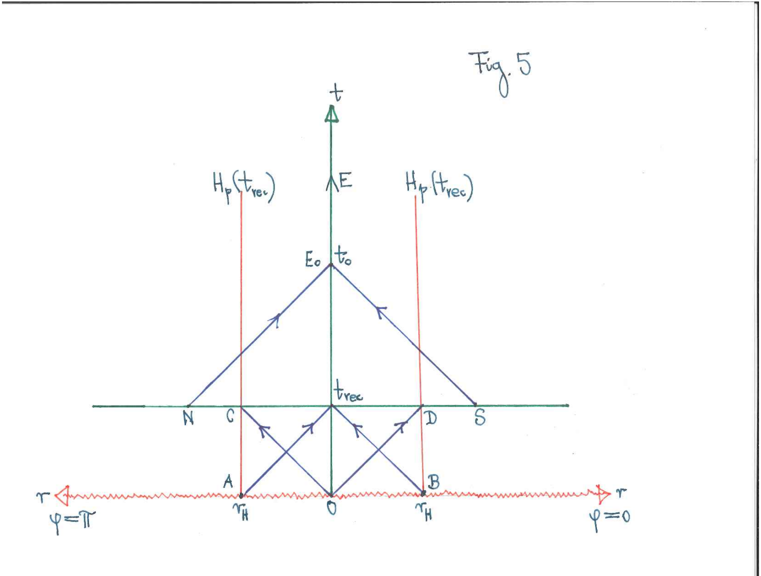
Assumption:

this process propagates from the initial place in all directions at $v \cong c$ and immediately smoothes out after arriving at any place.

It should smooth out Metagalaxy (the matter inside H_v) by recombination epoch. Is it possible?

Asumption:

the smoothing out process starts at $t \cong 0$ at $r = 0$ (E emerges from BB) and radially propagates.



Up to recombination $t = t_{\rm rec}$ space and matter are smoothed out within the light cone C0D.
C and D lie on $H_p(t_{\rm rec})$ and form the *causal limit sphere* at $t_{\rm rec}$.
Proof.
0C — radial outgoing null geodesic $\Rightarrow ds^2 = c^2\,dt^2 - R^2\,dr^2 = 0$,
$dt > 0 \Rightarrow dr > 0$ and

$$c\frac{dt}{R} = dr \Big| \int_0^{r_c} \Rightarrow r_C = r_D = c\int_0^{t_{\rm rec}} \frac{dt}{R} = r_H(t_{\rm rec}).$$

Diameter (proper distance) of the causal limit sphere = diameter of $H_p(t_{\rm rec})$ is

$$l_{CD} = 2d_H(t_{\rm rec}).$$

Radiation: $w = 1/3 \Rightarrow l_{CD} = 2 \cdot 2ct_{\rm rec}$.

We observe 2 antipodal points (poles N and S) on the celestial sphere at t_{rec} — on the last scattering sphere.
The diameter (proper distance) of LSS is $l_{NS} = 2 r_N R(t_{\mathrm{rec}})$.
$r_N = r_S$ — determined by the incoming radial null geodesic NE_0,

$$r_N = \int_{r_N}^{0} c \frac{dt(r)}{R} = c \int_{t_{\mathrm{rec}}}^{t_0} \frac{dt}{R}.$$

In galactic era: $p = 0 \Rightarrow R = a t^{2/3}$ for $k = 0 \Rightarrow$

$$r_N = \frac{3c}{a}(t_0^{1/3} - t_{\mathrm{rec}}^{1/3}) \Rightarrow l_{NS} = 6c\, t_{\mathrm{rec}} \left[\left(\frac{t_0}{t_{\mathrm{rec}}} \right)^{1/3} - 1 \right].$$

The ratio is

$$\frac{l_{NS}}{l_{CD}} = \frac{3}{2} \left[\left(\frac{t_0}{t_{\mathrm{rec}}} \right)^{1/3} - 1 \right].$$

For $t_0 \cong 13,7 \cdot 10^9$ y and $t_{\rm rec} \cong 5 \cdot 10^5$ y: $l_{NS}/l_{CD} \cong 45$.
LSS was covered with disks arising by intersections of LSS with causal limit spheres of local *particle horizons* at $t = t_{\rm rec}$. The number of these disks is

$$\frac{4\pi(\frac{1}{2}l_{NS})^2}{\pi d_H^2} = \left(\frac{l_{NS}}{d_H}\right)^2 \cong 4 \cdot 45^2 \cong 8000.$$

If signals at $v \cong c$ were emitted from a point Q at $t \approx 0$, all points in the causal limit sphere centered at Q and of radius $d_H = 2ct_{\rm rec}$ have been causally connected at $t = t_{\rm rec}$. Interiors of different spheres cannot interact up to times $t \gg t_{\rm rec}$.
Let the U. have R–W geometry since $t_g \approx 10^{-42}$ s — beginning of Grand Unification Era. Then the outcome $l_{NS}/l_{CD} \gg 1$ is *innocuous*.
But this has been regarded as *unnatural*.

The horizon problem:

the conjecture (early 1970's): *in GU Era the spacetime arised from Quantum Gravity Era highly irregular (a „generic" solution of EFE) and at* $t_g \approx 10^{-42}$ *s a smoothing process started at some point in space. Then up to* t_{rec} *only one disk on our LSS would be smoothed out to R–W geometry* $\Rightarrow$ *entire LSS would be still highly irregular — excluded by the isotropy of CMB radiation*.

Or: gravit. expansion does not give the early U. enough time to equilibrate.

Conclusion:

R–W geometry of the U. cannot be the outcome of a locally initiated smoothing process. This geometry must have been present in the whole space soon after BB.

How soon? Unclear.

Cosmology in R–W spacetime has a number of other problems. Two of them are most important.

1. The flatness problem („Dicke coincidence").

Observations $\Rightarrow \Omega_0 \cong 1 \Rightarrow k \cong 0$ — flat space. Observationally we shall never determine whether

$\Omega > 1$ — closed space S^3,

$\Omega < 1$ — hyperbolic (Lobatchevsky) open space H^3 or

$\Omega = 1$ — Euclidean E^3.

Critical density

$$\rho_c \equiv \frac{3H^2(t)}{8\pi G}, \qquad H(t) = \frac{\dot{R}}{R}, \qquad \Omega(t) \equiv \frac{\rho(t)}{\rho_c(t)}$$

$$\text{and Friedmann eq.} \qquad \frac{\dot{R}^2}{R^2} + \frac{k}{R^2} = \frac{8\pi G}{3}\rho$$

$$\text{may be recast to} \qquad \Omega - 1 = \frac{k}{H^2 R^2}.$$

Its modulus varies as $$\frac{d}{dt}|\Omega - 1| = -\frac{2\ddot{R}}{\dot{R}^3}.$$

Standard cosmology: $\dot{R} > 0$ and $\ddot{R} < 0 \Rightarrow \frac{d}{dt}|\Omega - 1| > 0$ — growing function.
In galactic era: $R = at^{2/3} \Rightarrow |\Omega - 1| = C_1 t^{2/3}$, at $t = t_{\rm rec}$

$$|\Omega(t_{\rm rec}) - 1| = \left(\frac{t_{\rm rec}}{t_0}\right)^{2/3} |\Omega_0 - 1|.$$

If $|\Omega_0 - 1| \cong 0,1 \Rightarrow |\Omega(t_{\rm rec}) - 1| \cong 10^{-4}$.
In the early U.: $R = b\sqrt{t} \Rightarrow |\Omega - 1| = C_2 t$ and

$$|\Omega(t) - 1| = \frac{t}{t_{\rm rec}}|\Omega(t_{\rm rec}) - 1| \approx 10^{-4}\frac{t}{t_{\rm rec}}.$$

For $t \approx 1$ s — beginning of radiation era (soon before $e^- e^+$ annihilation and BB nucleosynthesis): $|\Omega - 1| \approx 10^{-17}$.
These are *very special* initial conditions.

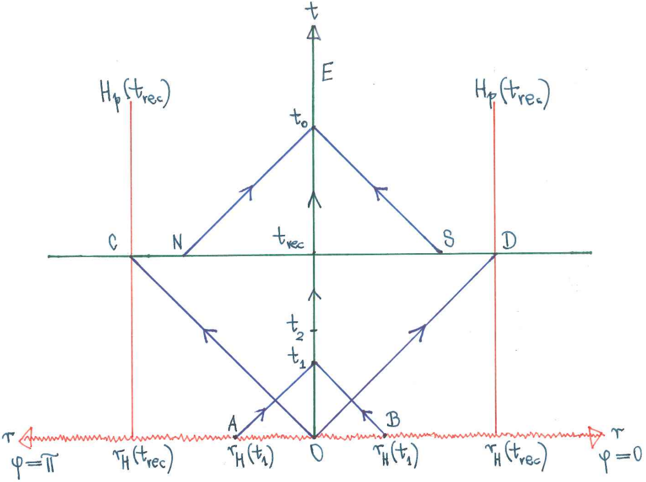
The flatness problem:
why is Ω_0 so close to 1? $\Leftrightarrow$ why is $\Omega(t = 1s) = 1 \pm 10^{-17}$?
If Ω_0 were 10^{-3} or 10^5, the U. would be a generic solution within R–W spacetimes and measurements of ρ_0 would be unambiguous.
The flatness problem is equivalent to: why is the U. so large and old?

2. The monopole problem („exotic–relics problem").
GUT: in Grand Unification Era ($E \approx T \approx 10^{16}$ GeV) **magnetic monopoles** were copiously created — heavy *stable* particles. They should survive up to now and dominate in the matter content of the U. (Ya. Zeldovich 1978)

Inflation solves the horizon problem

The horizon problem:
the size of LSS $\gg$ the diameter of $H_p(t_{\rm rec})$
is easily solved if
— for $t > t_{\rm rec}$ the evolution is standard,
— for $0 < t < t_{\rm rec}$ the expansion is much faster than t.
Then the diameter of $H_p(t_{\rm rec}) >$ the size of LSS.
Introduce an **inflationary epoch**: a short period in VEU of *very fast* (exponential) growth of $R(t)$ to make the radius $d_H(t_{rec})$ of H_p sufficiently large.
Assumptions on the early U. ($k = 0$):
a) for $0 < t < t_1$ matter is ultrarelat. $\Rightarrow R = at^{1/2}$,
b) for $t_1 < t < t_2$ the scale factor R grows Z times, $R(t_2) = ZR(t_1)$,
$Z \gg 10^8$, short period of fast expansion,
c) for $t_2 < t < t_{rec}$ matter is ultrarelat. $\Rightarrow R = bt^{1/2}$.
In galactic era matter is nonrelat. dust $\Rightarrow R = \gamma t^{2/3}$.
Fig. 6

Fig. 6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$r_H(t_{rec})$
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l_{NS} — diameter of LSS, $l_{NS} = 2R(t_{\rm rec})r_N$ and as before

$$l_{NS} = 6c\, t_{\rm rec} \left[\left(\frac{t_0}{t_{\rm rec}} \right)^{1/3} - 1 \right].$$

The diameter of $H_p(t_{\rm rec})$ is $l_{CD} = 2R(t_{\rm rec})r_H(t_{\rm rec})$ and

$$r_H(t_{rec}) = c \int_0^{t_1} \frac{dt}{R} + c \int_{t_1}^{t_2} \frac{dt}{R} + c \int_{t_2}^{t_{rec}} \frac{dt}{R}.$$

$t_2 - t_1$ — very short $\Rightarrow \int_{t_1}^{t_2}$ is neglected,

$$l_{CD} = 4ct_{rec} \left[(Z-1) \left(\frac{t_1}{t_{rec}} \right)^{1/2} + 1 \right].$$

$$\text{If} \quad Z \geq \frac{3}{2}(t_0^2 t_{rec} t_1^{-3})^{1/6} \gg 1 \quad \Rightarrow \quad \frac{l_{NS}}{l_{CD}} \leq 1.$$

t_1 — free parameter (model dependent), is limited by known physics and observations.
$t_1 \approx 1$ s (beginning of radiation era) $\Rightarrow Z > 1,5 \cdot 10^8$ — rather excluded (physics of the lepton era).
Typical model: $t_1 \approx 10^{-36}$ s, $t_2 \approx 10^{-32} s \Rightarrow Z > 10^{26}$.
Simplest inflationary evolution from t_1 to t_2:

$$R = Ae^{H(t-t_1)}, \quad A = \mathrm{const}, \quad H = \mathrm{const},$$

$$\Delta t = t_2 - t_1 \quad \Rightarrow \quad H = \frac{\ln Z}{\Delta t}.$$

Typical model:
$H \cong 2 \cdot 10^{53} \mathrm{km\,s^{-1} Mpc^{-1}}$ — huge numbers are characteristic for inflation (as for inflation in economics).

Inflation solves the flatness problem

The problem: why is $\Omega(t \approx 1s) = 1 \pm 10^{-17}$?

$$\Omega \text{ varies as} \qquad \frac{d}{dt}|\Omega - 1| = -\frac{2\ddot{R}}{\dot{R}^3}.$$

During inflation: $\dot{R} > 0$ and $\ddot{R} > 0 \Rightarrow |\Omega - 1|$ — decreasing function.
$\Omega(t_1)$ — very different from 1 — ,,generic" initial condition.
At the end of inflation:

$$R(t_2) = Ae^{H\Delta t} = Ae^{\ln Z} = AZ, \quad A = at_1^{1/2}, \text{then}$$

$$|\Omega(t_2) - 1| = \frac{kc^2}{H^2 R^2(t_2)} = \frac{(c\Delta t)^2}{A^2 (Z \ln Z)^2} \approx 10^{-110} \left(\frac{\text{km}}{A}\right)^2.$$

Unless A is extremely small, $|\Omega(t_2) - 1| \cong 0$.
$|\Omega - 1|$ grows for $t > t_2$ and reaches $\Omega = 1 \pm 10^{-17}$ for $t \approx 1$ s.

Inflation solves the monopole problem

Before inflation:

$$\rho = \frac{3}{32\pi G}\frac{1}{t^2} \qquad \text{universal (no arbitrary constants).}$$

From $t_1 \approx 10^{-36}$ s to $t_2 \approx 10^{-32}$ s, ρ decreases 10^8 times (standard).
If there were inflation:

$$\rho(t_2) = \frac{1}{Z^3}\,\rho(t_1) \approx 10^{-78}\,\rho(t_1)$$

— monopoles are diluted ⇒ now less than 1 monopole in Milky Way.
Problem:
ALL primordial matter is extremely diluted ⇒ now the U. should be nearly EMPTY.

Inflationary scenario

Before inflation ($0 < t < t_1$):
standard expansion driven by ultrarelat. primordial matter, $R \propto \sqrt{t}$.
Besides ordinary matter there is **inflaton field**.
Inflationary epoch ($t_1 < t < t_2$):
inflaton energy dominates and drives the accelerated expansion, $R \propto \exp(H(t - t_1))$. Matter and magnet. monopoles get diluted, all volumes get huge, space becomes almost empty. Close to $t = t_2$ inflaton field decays and its energy creates the Standard Model particles. These recurrent particles interact, get thermalized and form a dense hot plasma of all known elementary particles.
After the inflation terminates ($t > t_2$):
standard evolution of hot ultrarelat. plasma.
Now the U. is made of this secondary matter — the primordial matter is irrelevant as is greatly rarefied.

Such inflation does solve the flatness and exotic–relics (monopole) problems.
It is NOT a genuine solution to the horizon problem:
it *assumes* R–W geometry for all $t > 0$ and in R–W spacetime the horizon problem does *not* exist — matter and curvature are always and everywhere homogeneous and isotropic $\Rightarrow$ no smoothing out is necessary. („An elegant solution of a non–existent problem".)
Yet there is NO proof that in a generic spacetime emerging from initial singularity (BB), inflation will allow for smoothing out the irregularities before recombination $\Rightarrow$
the problem of why the U. has R–W spacetime remains unsolved.

Problems of physical inflation

Steven Weinberg, *Cosmology*, 2008:
,,So far, the details of inflation are unknown and the whole idea of inflation remains a speculation, though one that is increasingly plausible"
— mild and optimistic cricism.
The physical nature of inflation is vague.

1. What is inflaton?
In most models it is a scalar field. (Also vector fields have been applied.)
In the nature there is only 1 scalar — the **Higgs field**. The inflaton *cannot* be the Higgs field.
Inflaton — a new field beyond the Standard Model of elem. particles, introduced only for needs of cosmology — does *not* fit the known laboratory physics.

2. How is inflaton dynamically described?

Tenet of modern physics: all fundamental constituents of matter are quantum fields $\Rightarrow$ are described by QFT. Inflaton must be a quantum field.
Inflation (if it occurred) took place in a rapidly time varying curved spacetime.
QFT is well developed, fully consistent and firmly corroborated in flat Minkowski spacetime $\Rightarrow$ there is no satisfactory formulation of QFT in Friedmann spacetimes.
ALL models of inflation: its dynamics is calculated in various versions of *quasi–classical approximation* to QFT applied far beyond the validity domain of QFT $\Rightarrow$ reliability is unclear.
Actually:
— inflation is treated as a classical scalar field,
— gravitation $R(t)$ is accounted for only via the metric in eqs. of motion, $\eta_{\mu\nu} \to g_{\mu\nu}$.
Yet the quantum nature of inflaton is essential: its **quantum fluctuations** provide almost *scale–invariant density fluctuations* (observed in CMB radiation) — seeds of ,,structure" (galaxies).

3. Is the inflaton dynamics uniquely determined?

No. Beyond cosmology there are no indications on properties of inflation $\Rightarrow$ its self–interactions $\equiv$ potential may be *arbitrary* $\Rightarrow$ many models.

The fine–tuning problem:

inflation to work in cosmology requires a *very specific* potential — no motivation in known physics. This is why inflaton $\neq$ Higgs field.

The potential must allow for the **graceful exit** — to terminate the inflatory epoch and start the standard evolution driven by the recurrent matter.

Yet cosmology does *not* uniquely determine the potential $\Rightarrow$ multitude of models.

This s a strong objection:

inflaton potential seems to be an *ad hoc* contrivance to accommodate almost any data obtainable.

Benefits of inflation

Inflation is most valuable in that it predicts the initial conditions of formation of structures based on only 2 adjustable parameters:
— the spectral index and
— the amplitude of perturbations.
Conclusion:
this is a real strength of inflation and there is no real alternative to it for the origin of galaxies.

Criticism of inflation

Paul Steinhardt:
chaotic inflation $\Rightarrow$ **multiverse** — ,,it is a critial flaw in the inflationary paradigm".

Roger Penrose (*Road to Reality*, 2004):
inflation requires extremely specific initial conditions of its own — ,,There is something fundamentally misconceived about trying to explain the uniformity of the early U. as resulting from a thermalization process. [...] For, if the thermalization is actually doing anything [...] then it represents a definite increasing of the entropy. Thus, the U. would have been even more special before the thermalization than after."