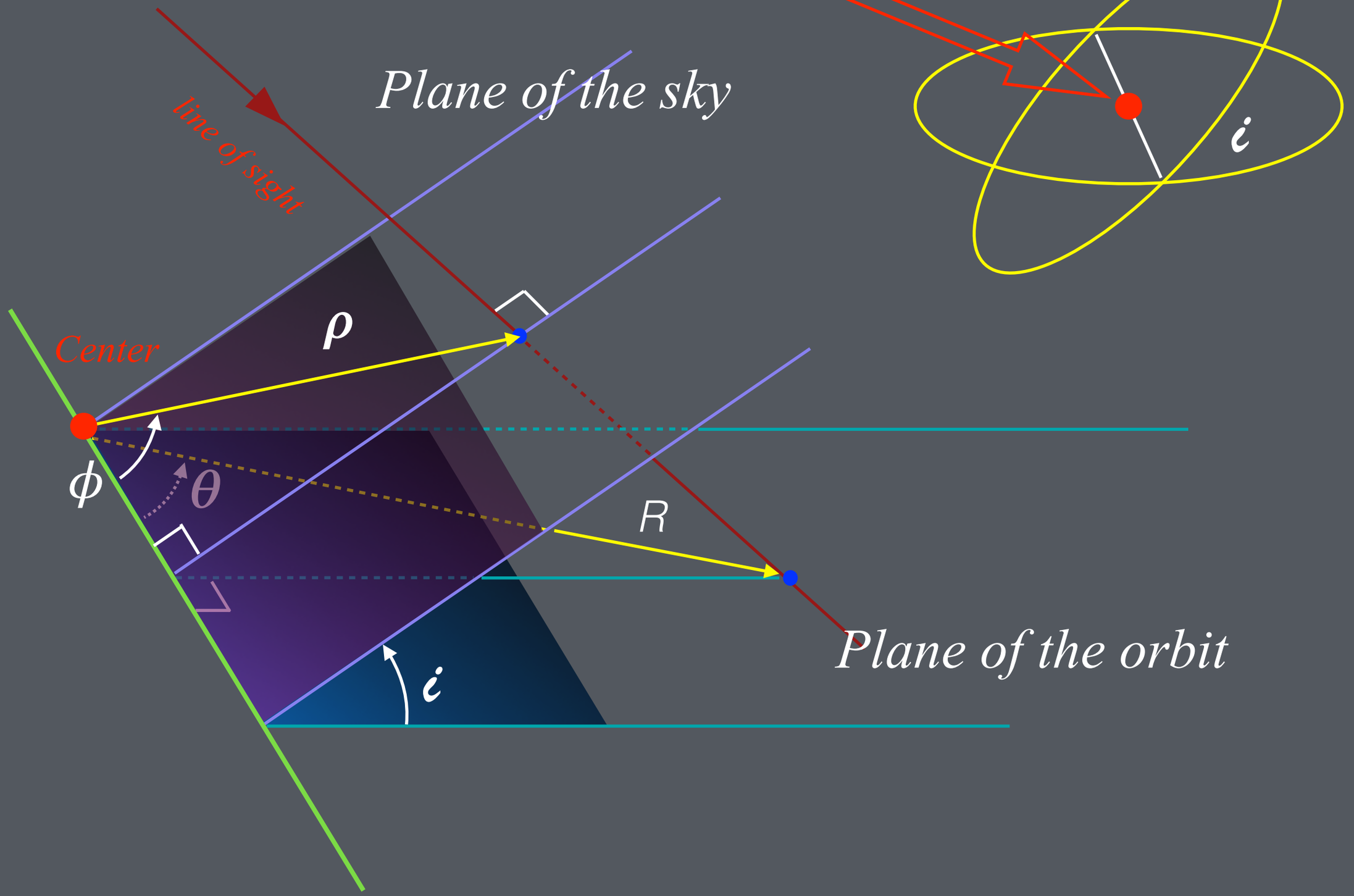
Mass of Galaxies
Lensing
&
Halo size

Krakow 2017 #2


Plane of the sky
line of sight
Center
ρ
ϕ
θ
R
i
Plane of the orbit
i

Binary galaxies

1) *It seems to be the way to go … however*
2) *Holmberg 1937 - Measure masses & for distances use distance indicators.*
3) *Page 1951 1962 - Not all selected sample - criticism especially by Turner 1976.*
4) *Chincarini 1971 - compact galaxies - very small & poor sample - Analysis following Page's procedure.*
5) *Turner 1976 - Inaccurate velocities - errors of 150 km /s and up to 450 km/s*
6) *White 1983 & others ……. see references therein*
7) *Not very much came out - In any case White et al.*

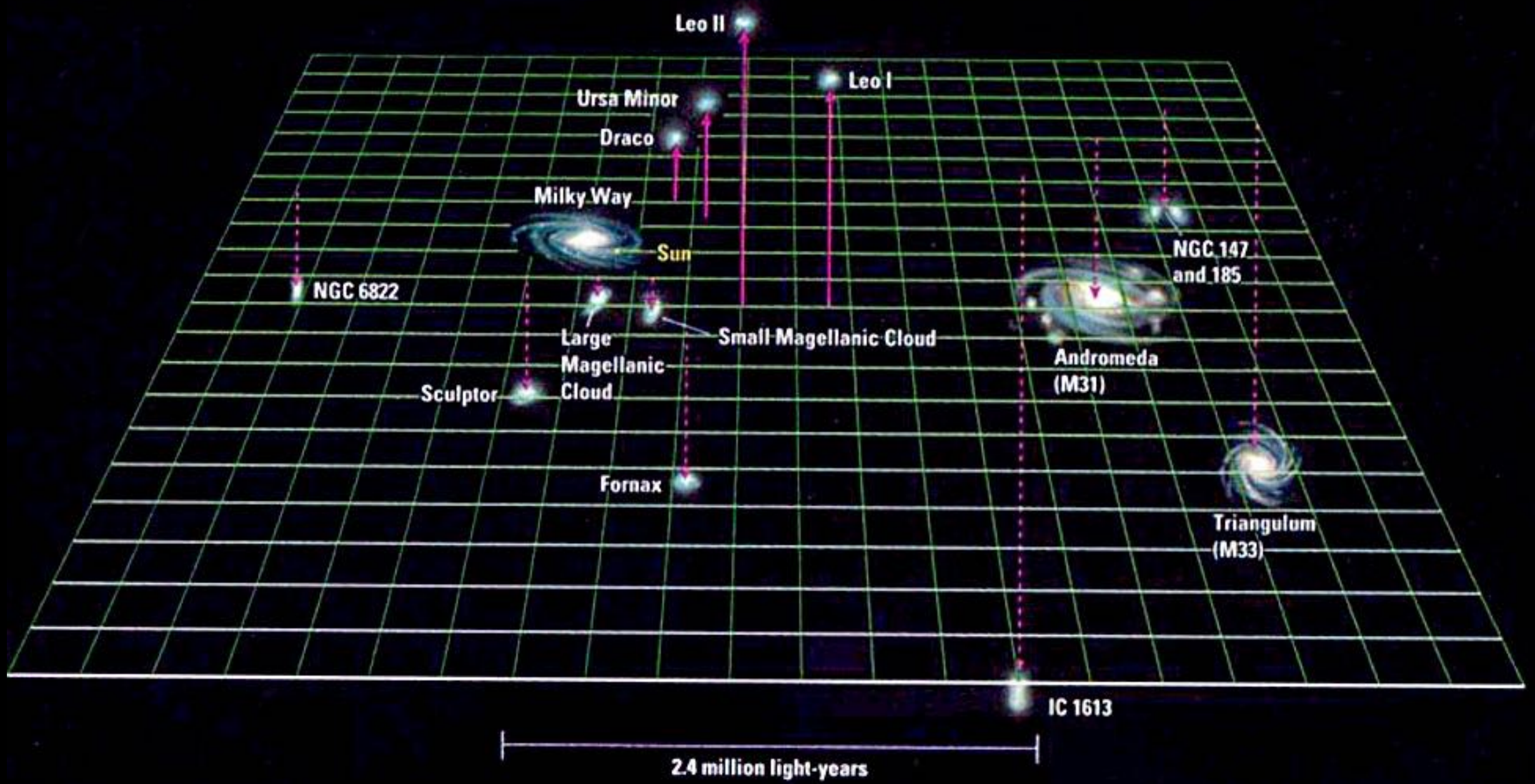
$$L^* = 3.4\,10^{10}\,h_{50}L_\odot$$

$$M/L = 47 \pm 13\left(r/_{100}\,kpc\right)\left(L/_{L^*}\right)^{-3/4} f(e)$$

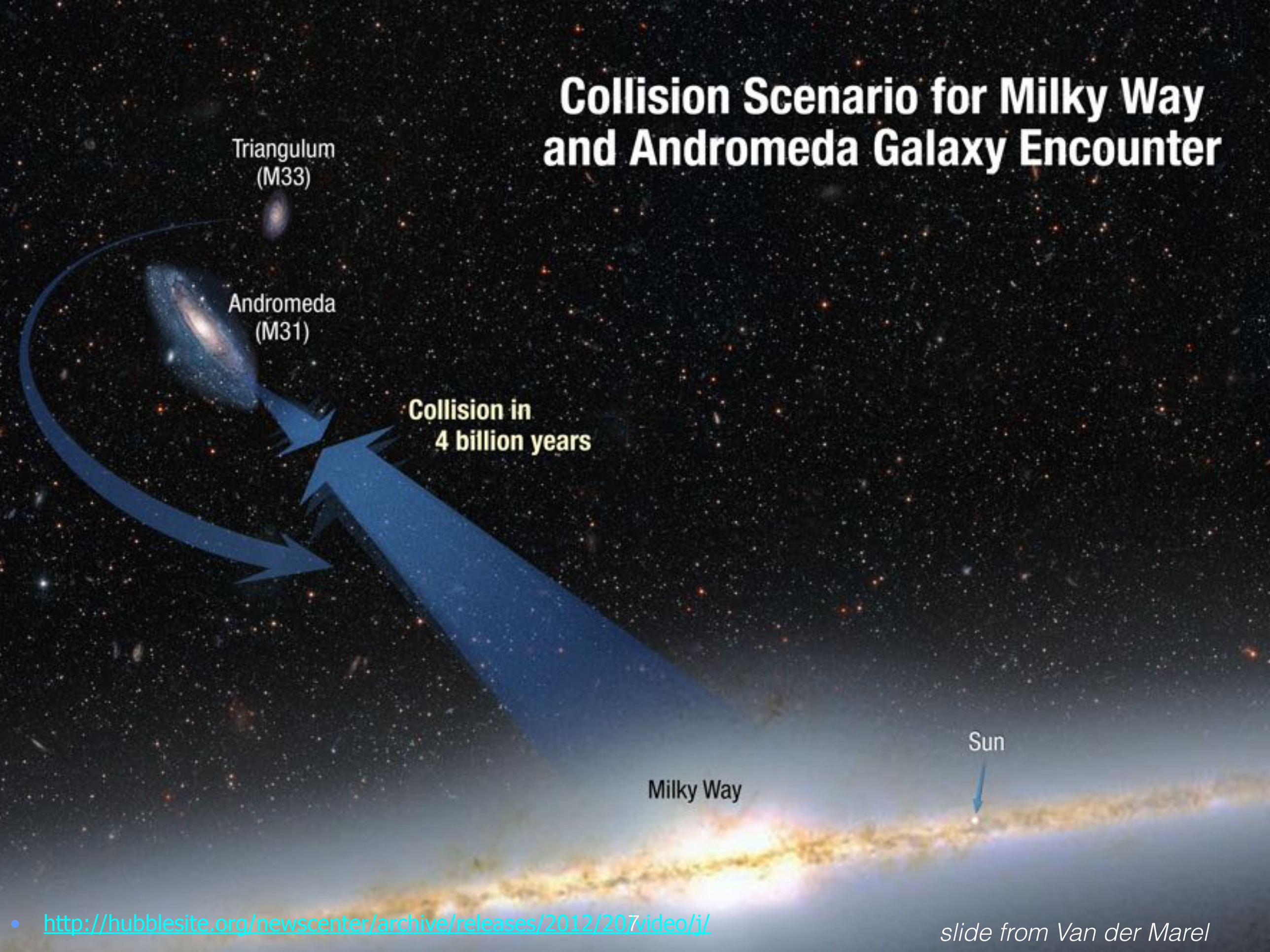
$$f(e) = 1 \text{ for circular orbits}$$

BINARIES
ARE FOR
COMPUTERS
on the other
hand

Local Group Of Galaxies



Collision Scenario for Milky Way and Andromeda Galaxy Encounter



- http://hubblesite.org/newscenter/archive/releases/2012/20/video/j/

slide from Van der Marel

0.000 billion years

Today

Following Kahn and Woltjer 1959

$$V_{(M31\text{-}MW)} = -125 \qquad Separation : 600kpc$$

$$\sim 10^{10}\ year\ to\ complete\ 1\ orbit$$

$$P^2 = \frac{4\,\pi^2\,a^3}{G\,(M_{MW} + M_{M\,31})} \leq 1\,10^{35}\ sec^2 \quad from\ which$$

$$M_{MW} + M_{M\,31} \geq 2\ 10^{12}\,M_{\odot}\ where$$

$$\left(M_{31} + M_{MW}\right)_{Schmidt} = 5\,10^{11} M_{\odot} \qquad \left(see\ next\right)$$

$$\text{Schmidt}\ (1956, 1957, 1965) \begin{cases} M_{M31} = 4\ 10^{11}\ M_{\odot} \\ M_{MW} = 1\ 10^{11}\ M_{\odot} \end{cases}$$

$$E_{Kin} = 1.25\ 10^{58}\ ergs$$

$$\Omega = -5.78\ 10^{57}\ ergs$$

The system should be exp*anding*

We need more massive galaxies
about 1. 5 10^{12} M_{sun} in the IGM
or mass spread out beyond the galaxy

Quite important the estimate of the IGM physical parameters

see Math presentation 4

Recent measurements

Watkins + al. 2010

$$M_{300\ kpc,\ MW} = 0.9 \pm 0.3\ 10^{12}\ M_{\odot}$$

$$M_{300\ kpc,\ M31} = 1.4 \pm 0.4\ 10^{12}\ M_{\odot}$$

Gonzales + al. 2010

$$M_{MW} + M_{M\,31} = 2.4^{+1.95}_{-1.05}\ 10^{12}\ M_{\odot}$$

$$M_{300\ kpc,\ M31} = 1.4 \pm 0.4\ 10^{12}\ M_{\odot}$$

Van der Marel et al. (2012) $M_{Local\ Group} = 3.17 \pm 0.57\ 10^{12}\ M_{\odot}$

The order of magnitude is the same as estimated by Kahn and Woltjer

The Deflection of light
&
Gravitational lensing

Do not Bodies act upon Light at a distance, and by their action bend its Rays; and is not this action strongest at the least distance? Opticks Newton 1704

Mitchel 1784 - Light would weaken getting out of a potential well

1784 - Henry Cavendish calculates deflection angle of corpuscular light ray in Newtonian gravity $-\delta = 2\,GM / Rc^2$.

Cavendish

1804 - von Soldner calculates deflection of 0.84 arcsecs for stars close to the limb of the sun. Value considered too small to be observationally relevant

Einstein deflection 1.75 arcsec

lensing Ellis

Einstein - 1911

4. Über den Einfluß der Schwerkraft auf die Ausbreitung des Lichtes; von A. Einstein.

Die Frage, ob die Ausbreitung des Lichtes durch die Schwere beeinflußt wird, habe ich schon an einer vor 3 Jahren erschienenen Abhandlung zu beantworten gesucht.[1]) Ich komme auf dies Thema wieder zurück, weil mich meine damalige Darstellung des Gegenstandes nicht befriedigt, noch mehr aber, weil ich nun nachträglich einsehe, daß eine der wichtigsten Konsequenzen jener Betrachtung der experimentellen Prüfung zugänglich ist. Es ergibt sich nämlich, daß Lichtstrahlen, die in der Nähe der Sonne vorbeigehen, durch das Gravitationsfeld derselben nach der vorzubringenden Theorie eine Ablenkung erfahren, so daß eine scheinbare Vergrößerung des Winkelabstandes eines nahe an der Sonne erscheinenden Fixsternes von dieser im Betrage von fast einer Bogensekunde eintritt.

Nach Gleichung (4) erleidet ein an einem Himmelskörper vorbeigehender Lichtstrahl eine Ablenkung nach der Seite sinkenden Gravitationspotentials, also nach der dem Himmelskörper zugewandten Seite von der Größe

$$\alpha = \frac{1}{c^2}\int_{\vartheta=-\frac{\pi}{2}}^{\vartheta=+\frac{\pi}{2}} \frac{k\,M}{r^2}\cos\vartheta\cdot ds = \frac{2\,k\,M}{c^2\,\Delta},$$

wobei k die Gravitationskonstante, M die Masse des Himmelskörpers, Δ den Abstand des Lichtstrahles vom Mittelpunkt des Himmelskörpers bedeutet. *Ein an der Sonne vorbeigehender Lichtstrahl erlitte demnach eine Ablenkung vom Betrage* $4\cdot 10^{-6}$ *= 0,83 Bogensekunden.* Um diesen Betrag er-

Einstein. Ann. d. Physik
1911 p. 908
Formula for deviation due
to Oé gravitational field
is
$$a = \frac{2kM}{c^2\Delta} = 0.''83 \text{ at limb}$$
$$\alpha \propto \frac{1}{\Delta}$$
at 1° from limb
$$a = 0.''28$$
at 6° from limb
$$a = 0.''06$$
It should be possible to
get this with some certainty
from several hundred stars

Curtis Lick Observ story

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ANGLO-AMERICAN TELEGRAPH Co. Ld.

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1923 APR 12 AM 6 37

ROYAL OBSERVATORY Q^x 3. 12 APR 1923 GREENWICH

JW 2PZ LD84 LICK OBSERVATORY CALIF 55 11/707P

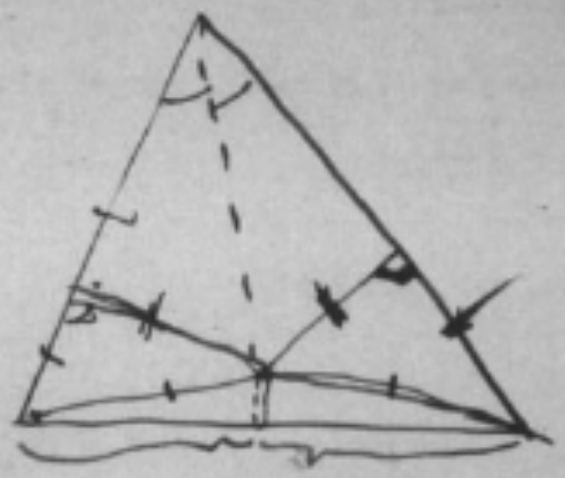
ASTRONOMER ROYAL GREENWICH

THREE PAIRS AUSTRALIA TAHITI ECLIPSE PLATES MEASURED BY CAMPBELL TRUMPLER SIXTY TWO TO EIGHTY FOUR STARS EACH FIVE OF SIX MEASUREMENTS COMPLETELY CALCULATED GIVE RESULTS BETWEEN ONE POINT FIFTY NINE AND ONE POINT EIGHTY SIX SECONDS ARC MEAN VALUE ONE POINT SEVENTY FOUR SECONDS WE NOT REPEAT EINSTEIN TEST NEXT ECLIPSE

CAMPBELL

Cable from Campbell to Dyson 12 April 1923, announcing preliminary results in favor of general relativity. (Courtesy Royal Greenwich Observatory archives, University of Cambridge.)

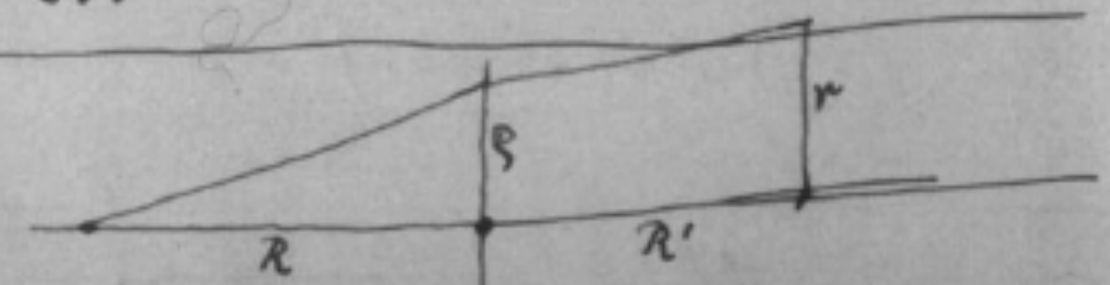
Alle Dreiecke sind gleichschenklig.



Neokestenberg

Berlin-Halensee,

Joachim Friedrichstr. 33.



$$r = \varrho \frac{R+R'}{R} - \frac{R'\alpha}{\varrho}$$

ϱ^{4r} nach unten negativ. Dann gilt auch für starkabgebeugte Strahl.

$$r_0 = \varrho_0 - \frac{1}{\varrho_0} \quad \ldots (1)$$

$$\boxed{\varrho_0^2 = \varrho^2 \frac{R+R'}{R R'\alpha}}$$

Einsteigl.

$$r = \cdot - \frac{R\alpha}{\varrho} = \cdot - \frac{R\alpha}{\varrho_0}\sqrt{\frac{R+R'}{R R'\alpha}}$$

$$= \cdot - \frac{1}{\varrho_0}\sqrt{\frac{R}{R'}(R+R')\alpha}$$

$$\left.\begin{aligned} \boxed{r_0 = r\sqrt{\frac{R}{R'(R+R')\alpha}}} \\ \boxed{\varrho_0 = \varrho\sqrt{\frac{R+R'}{R R'\alpha}}} \end{aligned}\right\} (2)$$

1) gibt zwei Wurzeln für ϱ_0

Von hier an Index p weggelassen.

$$2 + r^2 = \varrho^2 + \frac{1}{\varrho^2}$$

$$f = \varphi + \frac{\pi^2}{\varphi}$$

$$df = \left(1 - \frac{\pi^2}{\varphi^2}\right) d\varphi = \left(1 - \frac{1}{\varrho^4}\right) d\varphi$$

$$\mathcal{H}\, df = \pm H\, d\varphi$$

$$\mathcal{H} = \pm \frac{H}{1 - \frac{1}{\varrho^4}}$$

$$\mathcal{H}_{tot} = H\left\{\frac{1}{1 - \frac{1}{\varrho_1^4}} + \frac{1}{\frac{1}{\varrho_2^4} - 1}\right\} \quad \ldots (3)$$

Klammer gibt relative Helligkeit. (im ∞ = 1)

$$r = \frac{1}{x} - x$$

$$\{\ \} = \frac{1}{1 - x_1^4} + \frac{1}{x_2^4 - 1}$$

Einstein 1912 & later Science-1936 following

a request by Mandl

See also **Zwicky** 1937 -1956

LENS-LIKE ACTION OF A STAR BY THE DEVIATION OF LIGHT IN THE GRAVITATIONAL FIELD

Some time ago, R. W. Mandl paid me a visit and asked me to publish the results of a little calculation, which I had made at his request. This note complies with his wish.

The light coming from a star A traverses the gravitational field of another star B, whose radius is R_o. Let there be an observer at a distance D from B and at a distance x, small compared with D, from the extended central line $\overline{AB}$. According to the general theory of relativity, let α_o be the deviation of the light ray passing the star B at a distance R_o from its center.

" Provided that our present estimates of the masses of cluster nebulae are correct, the probability that nebulae which act as gravitational lenses will be found becomes practically a certainty."

Zwicky 1937

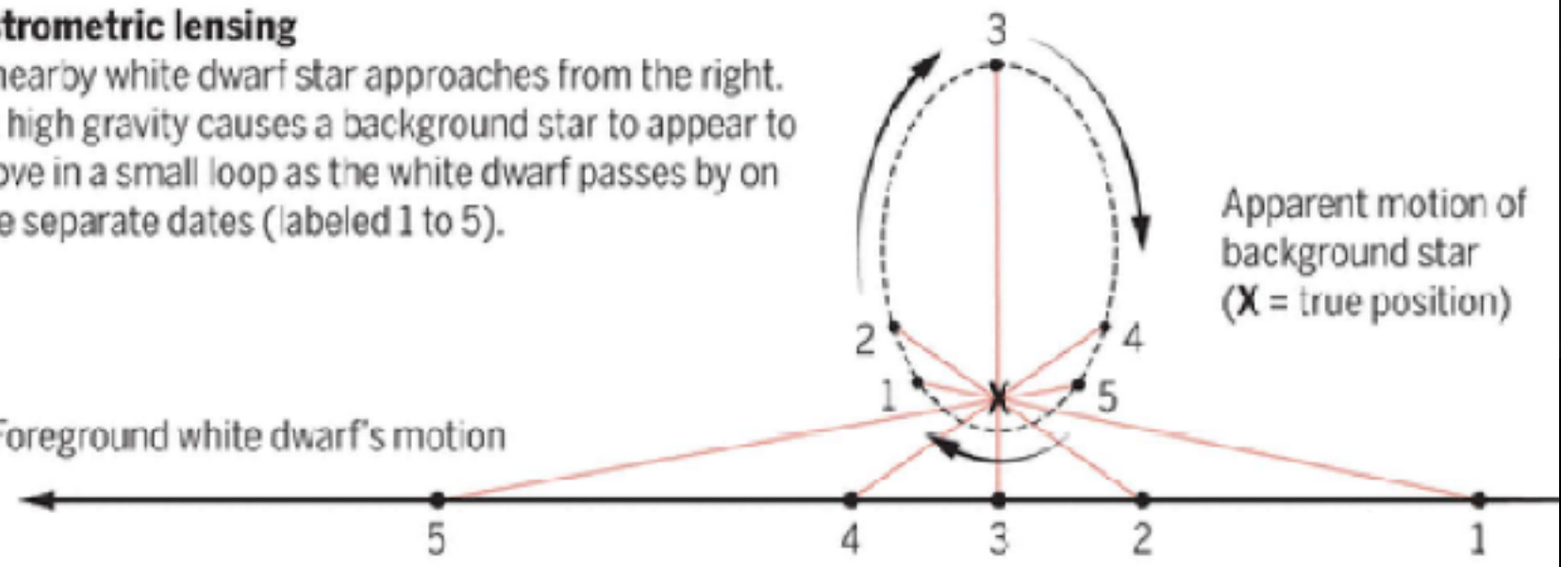
Sahu et al. 2017

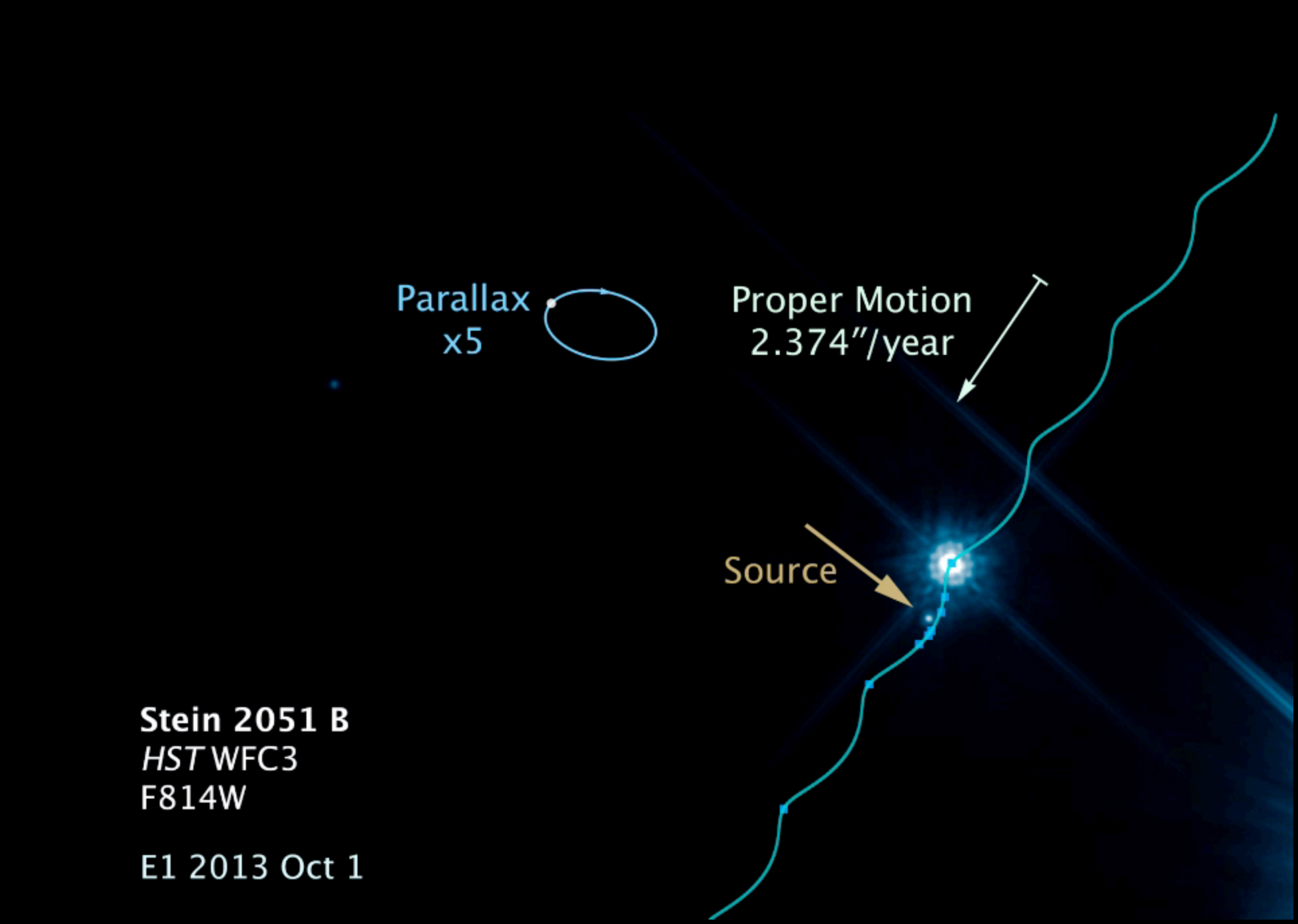
Gravitational Lensing by a Foreground White Dwarf

Foreground
White Dwarf

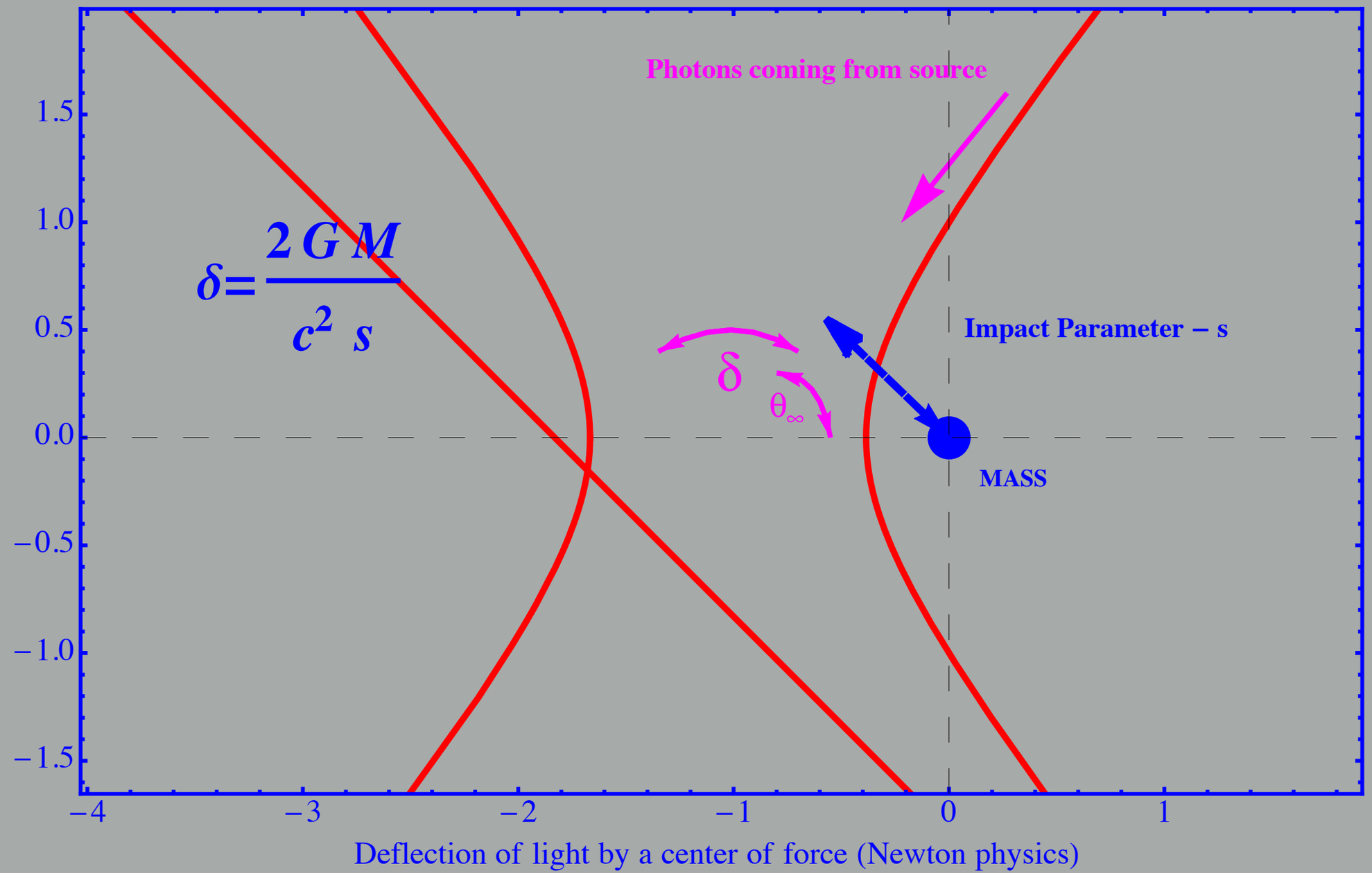
Background Star

Astrometric lensing
A nearby white dwarf star approaches from the right. Its high gravity causes a background star to appear to move in a small loop as the white dwarf passes by on five separate dates (labeled 1 to 5).

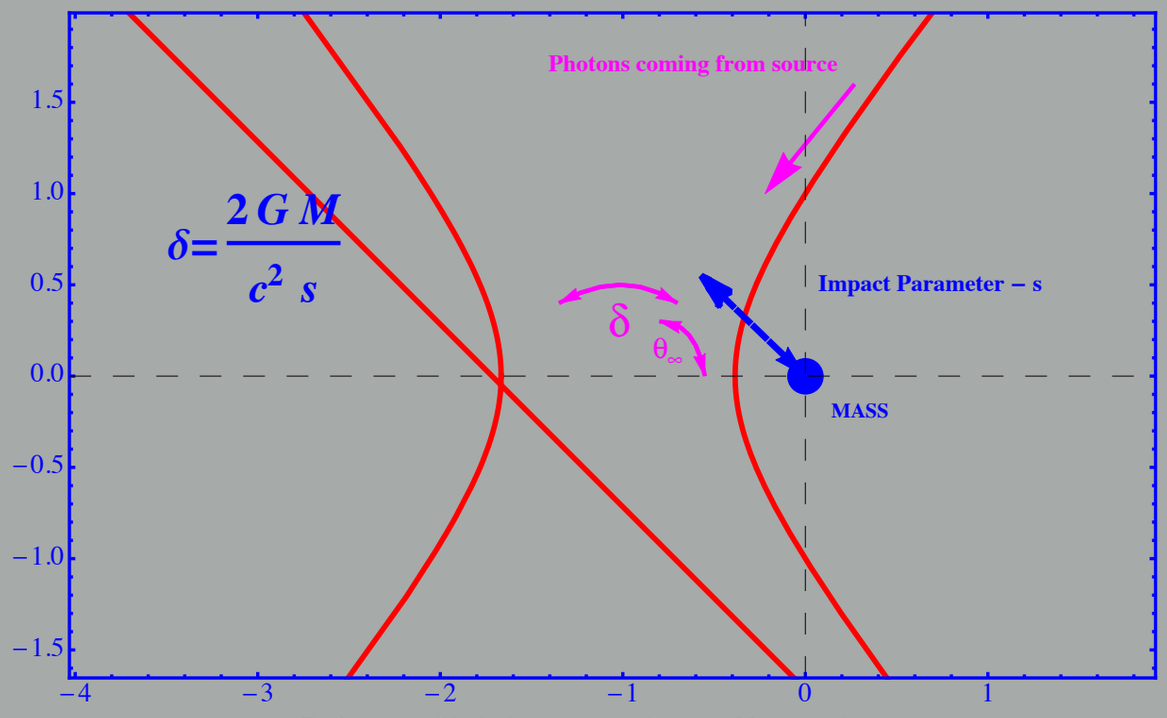



Parallax
x5
Proper Motion
2.374″/year
Source
Stein 2051 B
HST WFC3
F814W
E1 2013 Oct 1

For a change let's treat the deflection of light as a scattering problem in a central attractive force field.



Deflection of light by a center of force (Newton physics)



Deflection of light by a center of force (Newton physics)

$$f = -\frac{k}{r^2}\left(or\ f = \frac{Z\,Z' e^2}{r^2}\ for\ charged\ particles \right)$$

$$\frac{1}{r} = \frac{m\,k}{l^2}\left(1 - \varepsilon\cos\theta\right) with\ k = G\,M\,m \begin{bmatrix} orbit \\ hyperbolic \end{bmatrix}$$

&

$$l = m\,v\,s = s\sqrt{2\,m\,E}$$

equation of the orbit hyperbola

$$\varepsilon = \sqrt{1 + \frac{2\,E\,l^2}{m\,k^2}} = \sqrt{1 + \left(\frac{2\,E\,s}{k}\right)^2}$$

$$\frac{1}{r} = \frac{m\,k}{l^2}\left(1 - \varepsilon\cos\theta\right) \quad r \to \infty$$

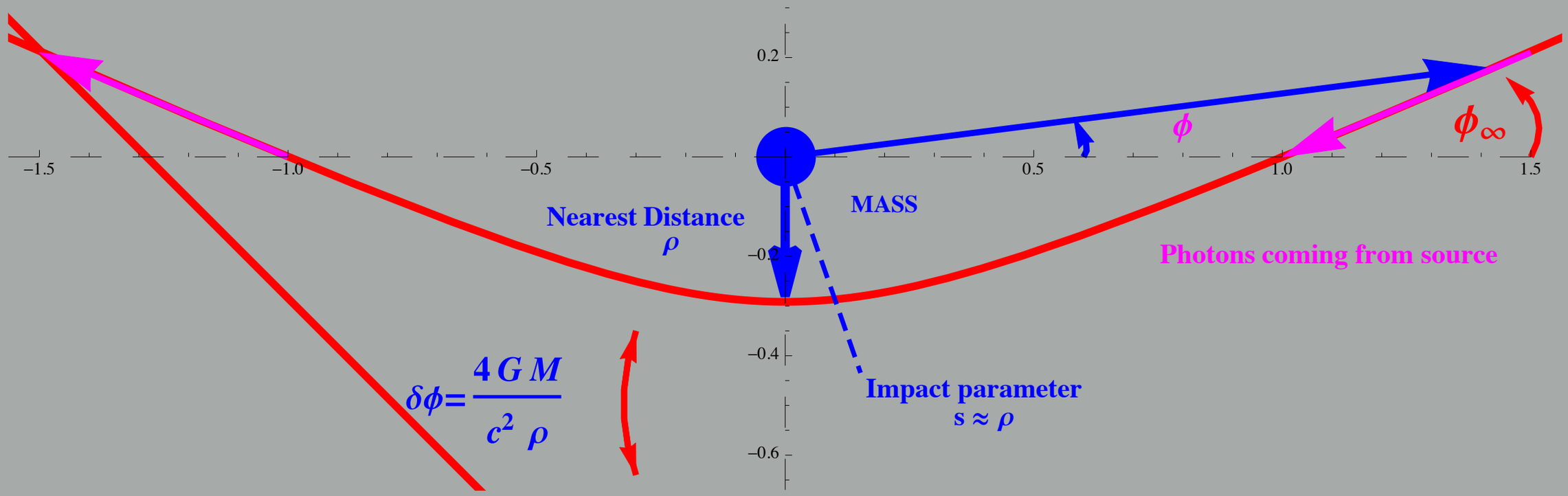
$$\cos\theta \to {}^{1}\!/_{\varepsilon} \quad \theta \to \theta_m \quad \cos\theta_m = {}^{1}\!/_{\varepsilon}$$

$$\varepsilon = \sqrt{1 + \left(\frac{2\,E\,s}{k}\right)^2} = \sqrt{1 + \left(\frac{2\,{}^{1}\!/_{2}\,m\,c^2 s}{G\,m\,M}\right)^2} \approx \frac{c^2 s}{G\,M}$$

$$\theta_m = Arc\,Cos\,\frac{1}{\varepsilon} \quad \delta = \pi - 2\,\theta_m = \pi - 2\,Arc\,Cos\,\frac{1}{\varepsilon} = \pi - 2\,Arc\,Cos\,\frac{G\,M}{c^2\,s}$$

$$\delta = \pi - \pi + 2\,\frac{G\,M}{c^2\,s} = \frac{2\,G\,M}{c^2\,s}$$

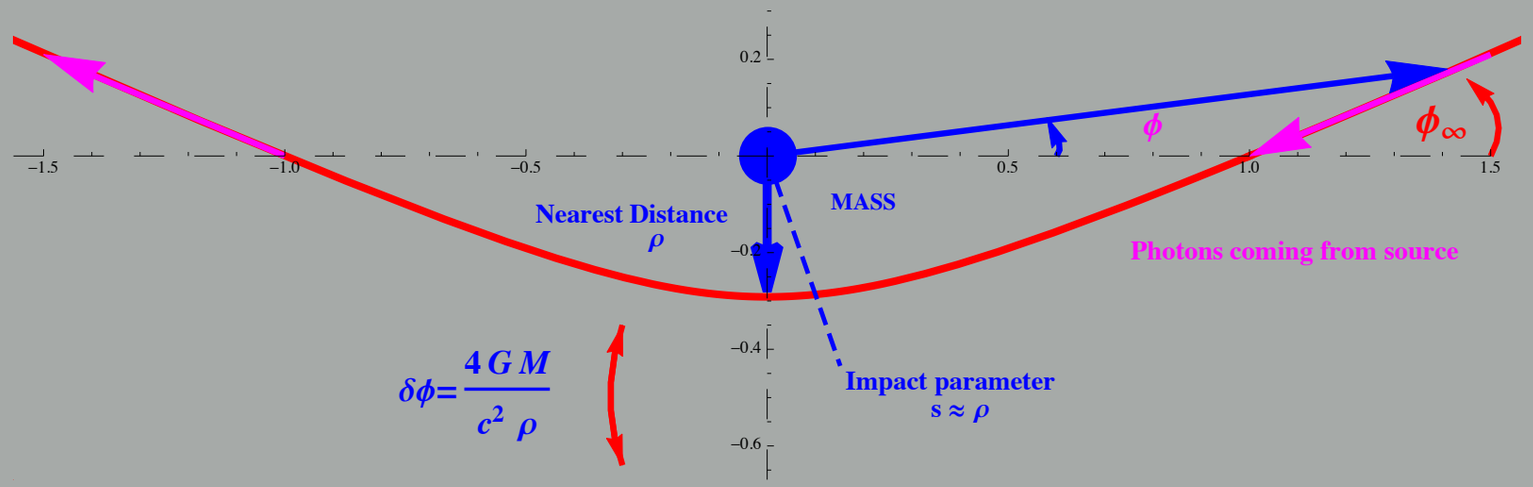
$$Arc\,\mathrm{Cos}\,x = \frac{\pi}{2} - \left[x + \frac{x^3}{2*3} + \ldots\ldots \right]$$



Hamilton – Jacobi equation

$$g^{ik}\frac{\partial S}{\partial x^i}\frac{\partial S}{\partial x^k}-m^2c^2=0 \qquad g^{ik}\frac{\partial \Psi}{\partial x^i}\frac{\partial \Psi}{\partial x^k}=0 \quad (m=0)$$

$$ds^2=\left(1-\frac{r_g}{r}\right)c^2\,dt^2-r^2\left(\sin^2\theta\,d\phi^2+d\theta^2\right)-\frac{dr^2}{1-\frac{r_g}{r}};\qquad r_g=\frac{2GM}{c^2}$$



Gravitational field
Central body

$$ds^2 = g_{ik}dx^i dx^k \qquad g^{ik}g_{kl} = \delta^i_l \qquad g^{ii} = 1/g_{ii}$$

$$ds^2 = \left(1 - \frac{r_g}{r}\right)c^2\,dt^2 - r^2\left(\sin^2\theta\,d\phi^2 + d\theta^2\right) - \frac{dr^2}{1 - \frac{r_g}{r}}; \qquad r_g = \frac{2\,G\,M_G}{c^2}$$

consider trajectory on a plane $\theta = \pi/2 \;\left(Sin\,\theta = 1\right)$ *so that* $d\theta = 0$

and

$$g^{00} = \left(1 - \frac{r_g}{r}\right)^{-1} \qquad g^{11} = g^{22} = \frac{1}{r^2} \qquad g^{33} = -\left(1 - \frac{r_g}{r}\right)$$

$$\left(1 - \frac{r_g}{r}\right)^{-1}\left(\frac{\partial\Psi}{c\,\partial t}\right)^2 - \left(1 - \frac{r_g}{r}\right)\left(\frac{\partial\Psi}{\partial r}\right)^2 - \frac{1}{r^2}\left(\frac{\partial\Psi}{\partial\phi}\right)^2 = 0$$

From Landau & Lifshitz Vol. 2

see Vol. 1
single particle

$$\Psi = -\omega_0 t + M\,\phi + \Psi_r(r) \quad \textit{defining} \quad \rho = {cM}/{\omega_0}$$

E_0
for a particle of mass m

$$\left(\frac{\partial \Psi}{c\,\partial t}\right) = -\frac{\omega_0}{c}\,; \quad \left(\frac{\partial \Psi}{\partial \phi}\right) = M = \frac{\rho\,\omega_0}{c}$$

$$\left(\frac{\partial \Psi_r}{\partial r}\right)^2 = \left(\frac{\partial \Psi}{\partial r}\right)^2 = \frac{1}{\left(1-\frac{r_g}{r}\right)}\left\{\frac{1}{\left(1-\frac{r_g}{r}\right)}\frac{\omega_0^2}{c^2} - \frac{M^2}{r^2}\right\}$$

$$\Psi_r(r) = {\omega_0}/{c}\int \sqrt{{r^2}/{\left(r-r_g\right)^2} - {M^2c^2}/{r^2\omega_0^2}\;\Big/\left(r-r_g\right)}\,dr$$

$$\Psi = -\omega_0 t + M\,\phi + {\omega_0}/{c}\int \sqrt{{r^2}/{\left(r-r_g\right)^2} - {M^2c^2}/{r^2\omega_0^2}\;\Big/\left(r-r_g\right)}\,dr$$

$$\Psi = -\omega_0 t + M\,\phi + \Psi_r(r)$$

Trajectory defined as

$$\frac{\partial \Psi}{\partial M} = \phi + \left(\frac{\partial \Psi_r}{\partial M}\right) = const$$

$$\partial\Psi/\partial M \Rightarrow \phi = \frac{\omega_0}{c}\int \frac{-2M\,\dfrac{c^2}{r^2\omega_0^2}\,\dfrac{r}{r-r_g}\,dr}{\int 2\sqrt{\dfrac{r^2}{(r-r_g)^2} - \dfrac{M^2c^2}{r^2\omega_0^2}\,\dfrac{r}{(r-r_g)}}}$$

This is the trajectory relating ϕ and r

$$\phi = \int \frac{dr}{r^2\sqrt{\dfrac{1}{\rho^2} - \dfrac{1}{r^2}\left(1-\dfrac{r_g}{r}\right)}}; \qquad r_g \equiv \frac{2\,G\,M_G}{c^2}$$

very small (~ 10^4 cm for the sun) so that $r_g \ll r$

$$\frac{1}{\left(1-\dfrac{r_g}{r}\right)^2} = \frac{r^2}{(r-r_g)^2} \Rightarrow 1 + \frac{2\,r_g}{r}$$

$$\Psi_r(r) = \omega_0/c \int \sqrt{\frac{r^2}{(r-r_g)^2} - \frac{M^2c^2}{r^2\omega_0^2}\,\frac{r}{(r-r_g)}}\,dr = \omega_0/c \int \sqrt{1 + \frac{2\,r_g}{r} - \frac{\rho^2}{r^2}}\,dr$$

The integral for $r_g \Rightarrow 0$

$$r = \rho/Cos\,\phi$$

straight line distance ρ from origin

$$\phi = \int \frac{dr}{r^2\sqrt{\dfrac{r^2-\rho^2}{r^2\rho^2}}} = \int \frac{dr}{\dfrac{r}{\rho}\sqrt{r^2-\rho^2}} = \frac{\rho}{\rho}\,arcos\,\frac{\rho}{r}$$

$$cos\,\phi = \frac{\rho}{r}$$

Non relativistic $r_g=0$

$$\Psi_r(r)=\omega_0/c\int\sqrt{1+\frac{2\,r_g}{r}-\frac{\rho^2}{r^2}}\,dr\,;\quad \Psi_r^{(0)}(r)=\omega_0/c\int\sqrt{1-\frac{\rho^2}{r^2}}\,dr$$

$$\Psi_r(r)=\Psi_r^{(0)}+r_g\,\omega_0/c\int\frac{\frac{r_g}{r}}{\sqrt{1+\frac{\rho^2}{r^2}}}\,dr=\Psi_r^{(0)}+r_g\,\omega_0/c\int\frac{1}{\sqrt{r^2-\rho^2}}\,dr$$

$$\Psi_r=\Psi_r^{(0)}+r_g\,\omega_0/c\,\cosh^{-1}\,r/\rho$$

Develop powers r_g/r

Photons go from large distance R to the point $r=\rho$ and back to large distance R

$$\Delta\Psi_r=\Delta\Psi_r^{(0)}+2\frac{r_g\omega_0}{c}\cosh^{-1}\frac{r}{\rho}$$

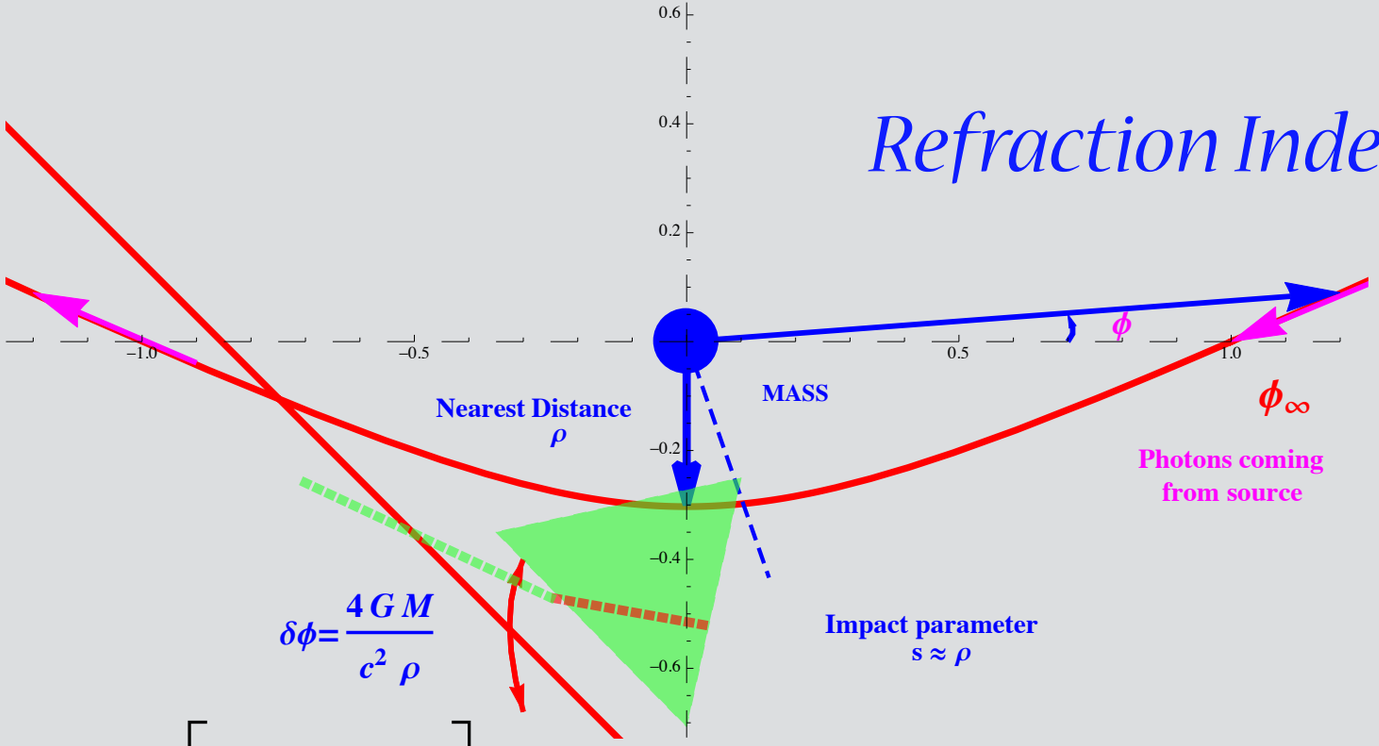
$$\phi+\left(\frac{\partial\Psi_r}{\partial M}\right)=const\quad\Rightarrow\Delta\phi=-\frac{\partial\Delta\Psi_r}{\partial M}=-\frac{\partial\Delta\Psi_r^{(0)}}{\partial M}+\frac{2r_gR}{\rho\sqrt{R^2-\rho^2}}$$

$$R->\infty\quad straight\ line\ \Delta\phi=\pi\Rightarrow\Delta\phi=\pi+\frac{2\,r_g}{\rho}$$

Bending

$$\delta\,\phi=\frac{2\,r_g}{\rho}=\frac{4\,G\,M}{c^2\rho}$$

Refraction Index



$$n = 1 - \frac{2}{c^2}\Phi = 1 + \frac{2}{c^2}\left|\Phi\right|$$

$$vel = c/n$$

$l \approx r$ *deflection angle very small*

$$Series\left[\frac{c}{1+2/c^2\left|\Phi\right|}\right] = c - \left|2\Phi\right|/c + \left|4\Phi^2\right|/c^3 + \ldots.. \approx c - \left|2\Phi\right|/c$$

$$vel = \frac{dl}{dt} \Rightarrow dt = \frac{dl}{vel} = \frac{dl}{c - \left|2\Phi\right|/c} \approx 1/c + \left|2\Phi\right|/c^3$$

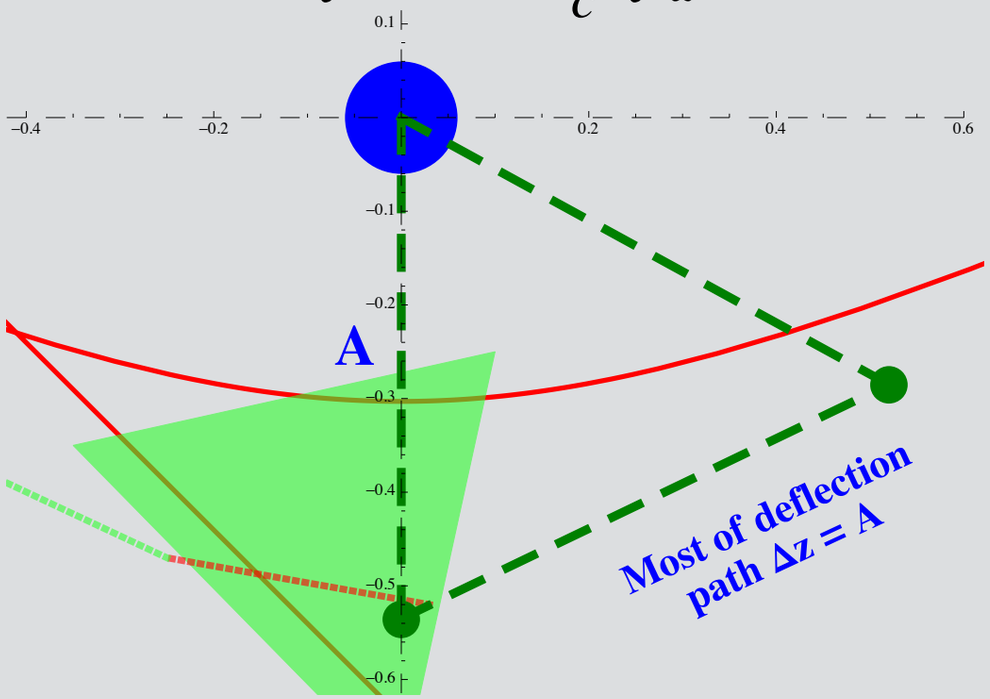
$$\Delta t = \int_{observer}^{source} 1/c + \left|2\Phi\right|/c^3\, dl = \int_{observer}^{source} \left|2\Phi\right|/c^3\, dl$$

$$Series\left[\frac{1}{c - \left|2\Phi\right|/c}\right] = 1/c + \left|2\Phi\right|/c^3 + \ldots..$$

$$\delta\phi = \int \nabla_\perp n\, dl = \frac{2}{c^2}\int_{-\infty}^{\infty} \nabla_\perp \left|\Phi\right| dl$$

$$\Phi(A,z) = -\frac{GM}{\left(A^2+z^2\right)^{1/2}} \qquad A \sim \rho \sim s$$

$$\nabla_\perp \Phi(A,z) = \frac{GM\ 1/2\left(A^2+z^2\right)^{-1/2}}{\left(A^2+z^2\right)} 2A = \frac{G\ M\ A}{\left(A^2+z^2\right)^{3/2}}$$



my ref. Theory Harvard

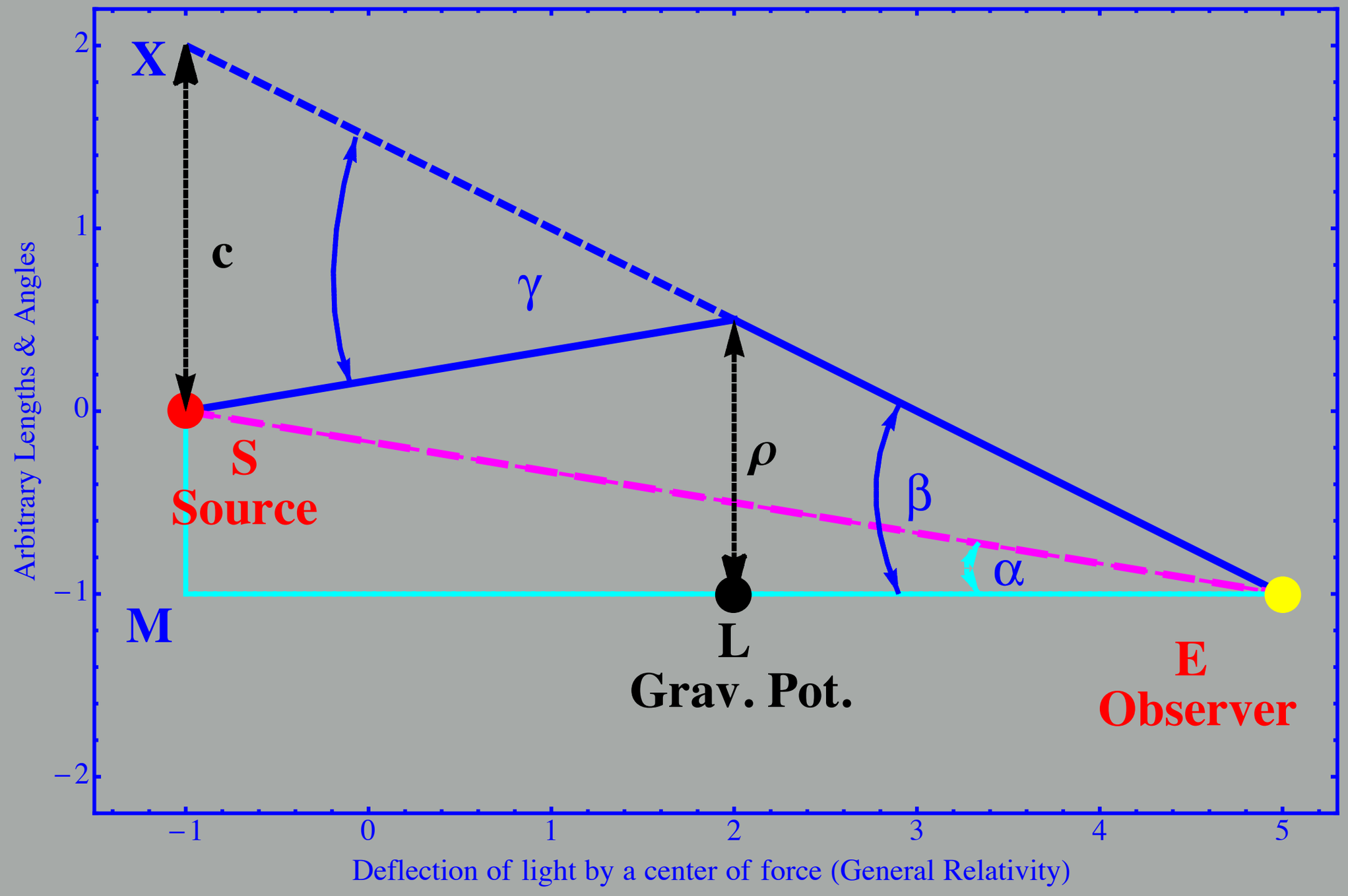
$$\delta\phi = \int \nabla_{\perp} n \, dl = \frac{2}{c^2} \int_{-\infty}^{\infty} \nabla_{\perp} \left| \Phi \right| dl$$

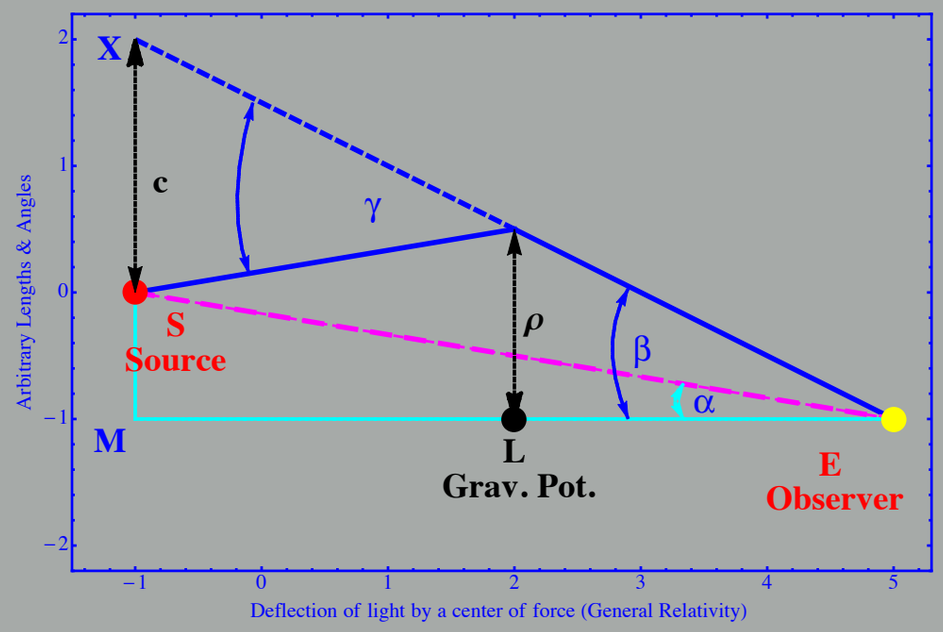
$$\delta\phi = \frac{2}{c^2} \int_{-\infty}^{\infty} \frac{G \; M \; A}{\left(A^2 + z^2 \right)^{3/2}} \, dz = \frac{2}{c^2} 2 \; G \; M \; A \int_{0}^{\infty} \frac{1}{\left(A^2 + z^2 \right)^{3/2}} \, dz =$$

$$\frac{4}{c^2} G \; M \; A \int_{0}^{\infty} \frac{1}{\left(A^2 + z^2 \right)^{3/2}} \, dz = \frac{4}{c^2} G \; M \; A \left| \begin{matrix} \infty \\ 0 \end{matrix} \right. \frac{z}{A^2 \left(A^2 + z^2 \right)^{1/2}}$$

$$since \quad \left\{ limit_{z \to \infty} \frac{z}{\left(A^2 + z^2 \right)^{3/2}} = \frac{1}{A^2} \right.$$

$$= \frac{4 \; G \; M \; A}{A^2 c^2} = \frac{4 \; G \; M}{A \, c^2}$$


X
c
γ
S
Source
ρ
β
α
M
L
Grav. Pot.
E
Observer
Arbitrary Lengths & Angles
Deflection of light by a center of force (General Relativity)
2
1
0
−1
−2
−1
0
1
2
3
4
5



I can write the lens equation as

$$\beta = \alpha + \frac{\beta_E^2}{\beta}\,;\ \beta^2 - \alpha\,\beta - \beta_E^2 = 0$$

$$\beta_{\pm} = \frac{\alpha \pm \sqrt{\alpha^2 + 4\beta_E^2}}{2}$$

$K = 0$ *Universe & Angular Diameter Distance*

$$\delta\phi = \gamma = \frac{4\,G\,M}{c^2\,\rho} \qquad \rho = EL\,\beta$$

$$EM\ \beta = EM\ \alpha + LM\ \gamma \qquad EM\,(\beta - \alpha) = LM\ \gamma$$

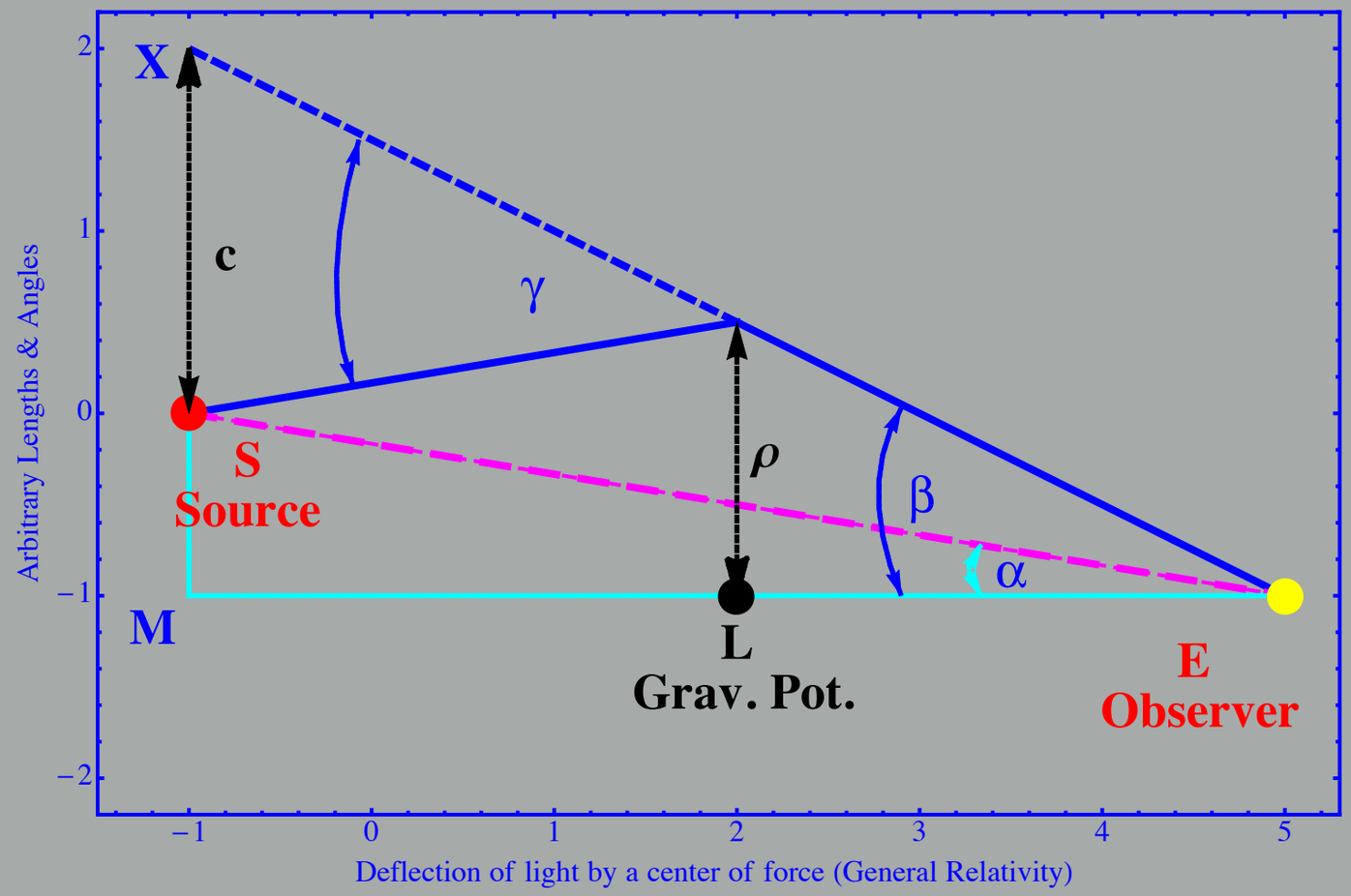
$$\gamma = \frac{EM\,(\beta - \alpha)}{LM} = \frac{4\,G\,M_G}{c^2\ EL\,\beta}$$

$$\beta - \alpha = \frac{4\,G\,M_G}{c^2\ \beta}\,\frac{LM}{EM\ EL}$$

$$\beta_E^2 \equiv \frac{4\,G\,M_G}{c^2}\,\frac{LM}{EM\ EL} \qquad \Rightarrow (\beta - \alpha)\,\beta = \beta_E^2$$

Measuring the mass

- *Measure only separation images*
- $\left|\beta_+ - \beta_-\right|^2 \geq 4\beta_E^2$ *limit on mass*
- *Lens visible* α *unknown*
- *use* $\beta_+\beta_- = -\beta_E^2 \Rightarrow$ *Mass*



$$\begin{cases} Observed\ \beta_+ - \beta_- = 1'' \\ \alpha\ not\ observed \\ ES \sim EM = 200\ Mpc \\ EL = 100\ Mpc \\ LM = 100\ Mpc \end{cases}$$

$$\beta_\pm = \frac{\alpha \pm \sqrt{\alpha^2 - 4\beta_E^2}}{2} \qquad \beta_+ - \beta_- = \sqrt{\alpha^2 - 4\beta_E^2} \qquad \left|\beta_+ - \beta_-\right|^2 = \left(\alpha^2 - 4\beta_E^2\right)$$

$$\left|\beta_+ - \beta_-\right|^2 \geq 4\beta_E^2 \qquad 1'' \geq 4\,\frac{4\,G\,M_G}{c^2}\,\frac{LM}{EM\ EL}$$

$$M_G \leq 6.1\,10^9\ M_\odot$$

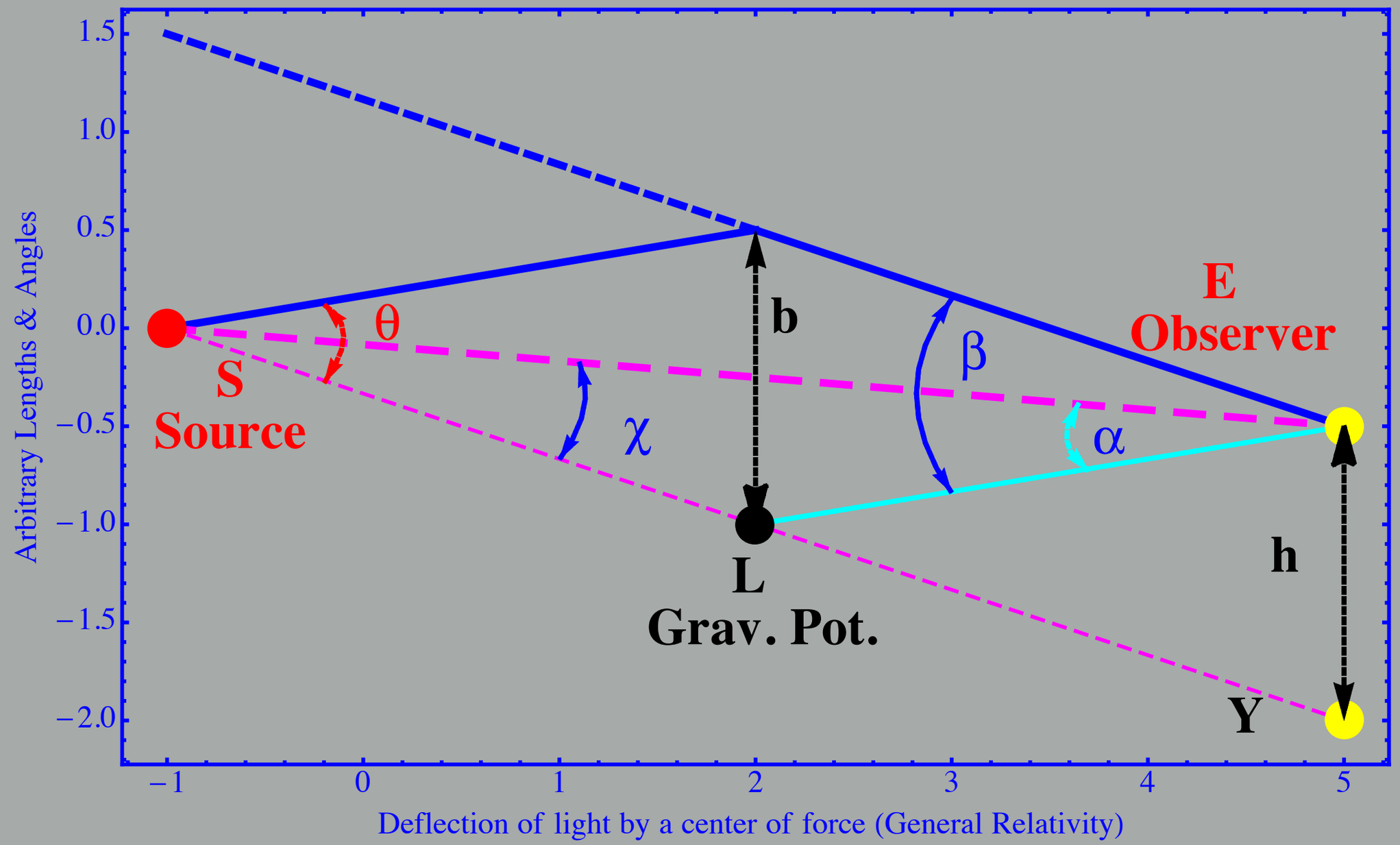
Easy to remember

$$\theta_E^2 = \frac{4\,G\,M_G}{c^2}\frac{LM}{EM\;EL}$$

$$\frac{LM}{EM\;EL} \sim \frac{ES-EL}{EL\;ES} \quad \textit{Define} \quad \textit{Reduced Distance to lens as}$$

$$\frac{1}{RDL} = \frac{1}{EL} - \frac{1}{ES} \Rightarrow RDL = \frac{EL\;ES}{ES-EL}$$

$$\theta_E = \sqrt{\frac{4\,G\,M_G}{c^2}\frac{1}{RDL}}$$


1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−2.0
−1
0
1
2
3
4
5
Arbitrary Lengths & Angles
Deflection of light by a center of force (General Relativity)
S
Source
θ
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Observer
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h
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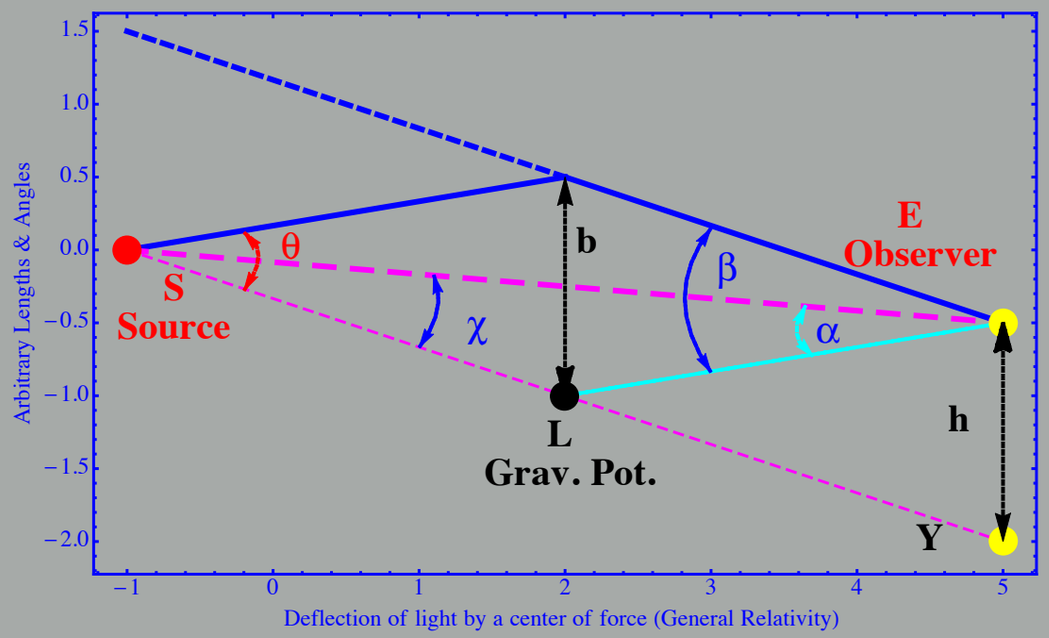
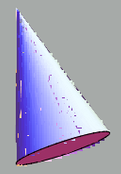




$$d\Omega = \sin\theta\, d\theta\, d\phi \approx \theta\, d\theta\, d\phi$$

$$\text{to a distant observer } \frac{L}{4\pi}\,\theta\, d\theta\, d\phi$$

Detector with h dh dϕ area receives

$$l = \frac{L}{4\pi}\,\theta\, d\theta\, d\phi \frac{1}{h\, dh\, d\phi \left(1+z_s\right)^2} =$$

$$= \frac{L}{4\pi}\frac{1}{\left(1+z_s\right)^2}\frac{\theta\, d\theta}{h\, dh} = l_0\, SE^2 \frac{\theta\, d\theta}{h\, dh}$$

$$\text{since}\left(\text{no lens}\right) \quad l_0 = \frac{L}{4\pi}\frac{1}{\left(1+z_s\right)^2 SE^2}$$

$$\frac{\chi}{\alpha} = \frac{EL}{SL}; \quad SL\,\theta = b = EL\,\beta$$

$$h = SE\,\chi = SE\,\frac{EL}{SL}\,\alpha \quad dh = SE\,\frac{EL}{SL}\,d\alpha$$

$$\theta = \frac{b}{SL} = \frac{EL\,\beta}{SL} \quad d\theta = \frac{EL}{SL}d\beta \qquad \frac{\theta\, d\theta}{h\, dh} = \left(\frac{EL}{SL}\right)^2 \beta d\beta \left(\frac{SL}{SE\ EL}\right)^2 \frac{1}{\alpha\, d\alpha}$$

$$\frac{\theta\, d\theta}{h\, dh} = \frac{1}{SE^2}\frac{\beta d\beta}{\alpha\, d\alpha}$$

$$l = l_0\, SE^2 \frac{\theta\, d\theta}{h\, dh} = l_0 \frac{\beta d\beta}{\alpha\, d\alpha}$$

$$\begin{cases} \beta^2-\alpha\,\beta-\beta_E^2=0 \quad \beta_\pm=\dfrac{\alpha\pm\sqrt{\alpha^2-4\beta_E^2}}{2} \quad \dfrac{l}{l_0}=\dfrac{\beta\,d\beta}{\alpha\,d\alpha} \\ \alpha=\beta-\dfrac{\beta_E^2}{\beta} \quad d\alpha=d\beta+\dfrac{\beta_E^2}{\beta^2}d\beta=d\beta\left(1+\dfrac{\beta_E^2}{\beta^2}\right) \\ \alpha=\beta\left(1-\dfrac{\beta_E^2}{\beta^2}\right) \quad \alpha\,d\alpha=\beta\left(1-\dfrac{\beta_E^2}{\beta^2}\right)d\beta\left(1+\dfrac{\beta_E^2}{\beta^2}\right) \\ \dfrac{\alpha\,d\alpha}{\beta\,d\beta}=1-\beta_E^4\big/\beta^4 \quad l=\dfrac{l_0}{1-\beta_E^4\big/\beta^4} \end{cases}$$

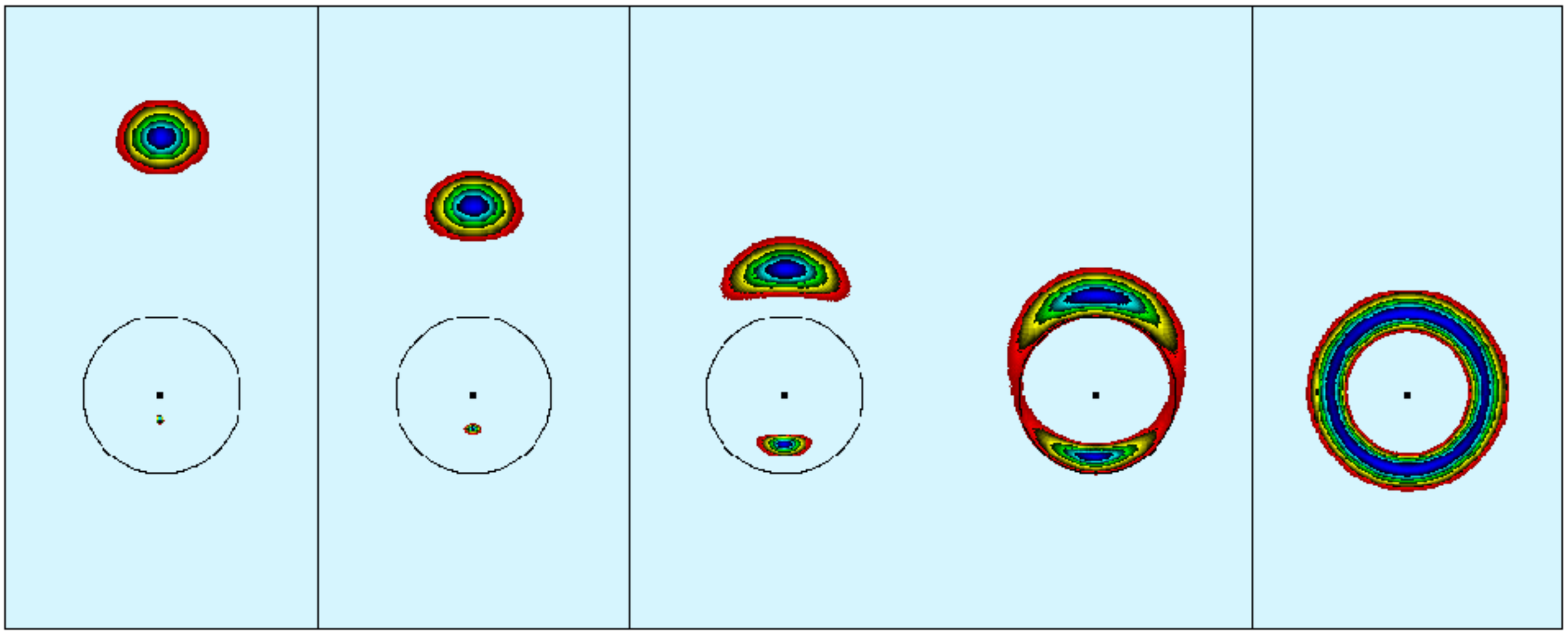
$$\begin{cases} \beta_E\gg\alpha \quad \beta_\pm\to\pm\beta_E+{}^1\!/_2\,\alpha \\ l=l_0\dfrac{\beta_E}{2\,\alpha}\begin{cases} \textit{Series see MATH} \\ \textit{or}\ \dfrac{\beta\,d\beta}{\alpha\,d\alpha}=\dfrac{\beta\,1/2\,d\alpha}{\alpha\,d\alpha} \end{cases} \\ \beta_E\ll\alpha\begin{cases} \beta_+ -> \alpha\gg\beta_E;\ \beta_E^4\big/\beta^4 -> \beta_E^4\big/\alpha^4 -> 0 \\ l=l_0 \\ \beta_-=\dfrac{\beta_E^2}{\alpha}\ll\beta_E\equiv\textit{Not visible} \\ (\textit{Series see MATH}) \end{cases} \end{cases}$$

See plot for α and distance from the lens

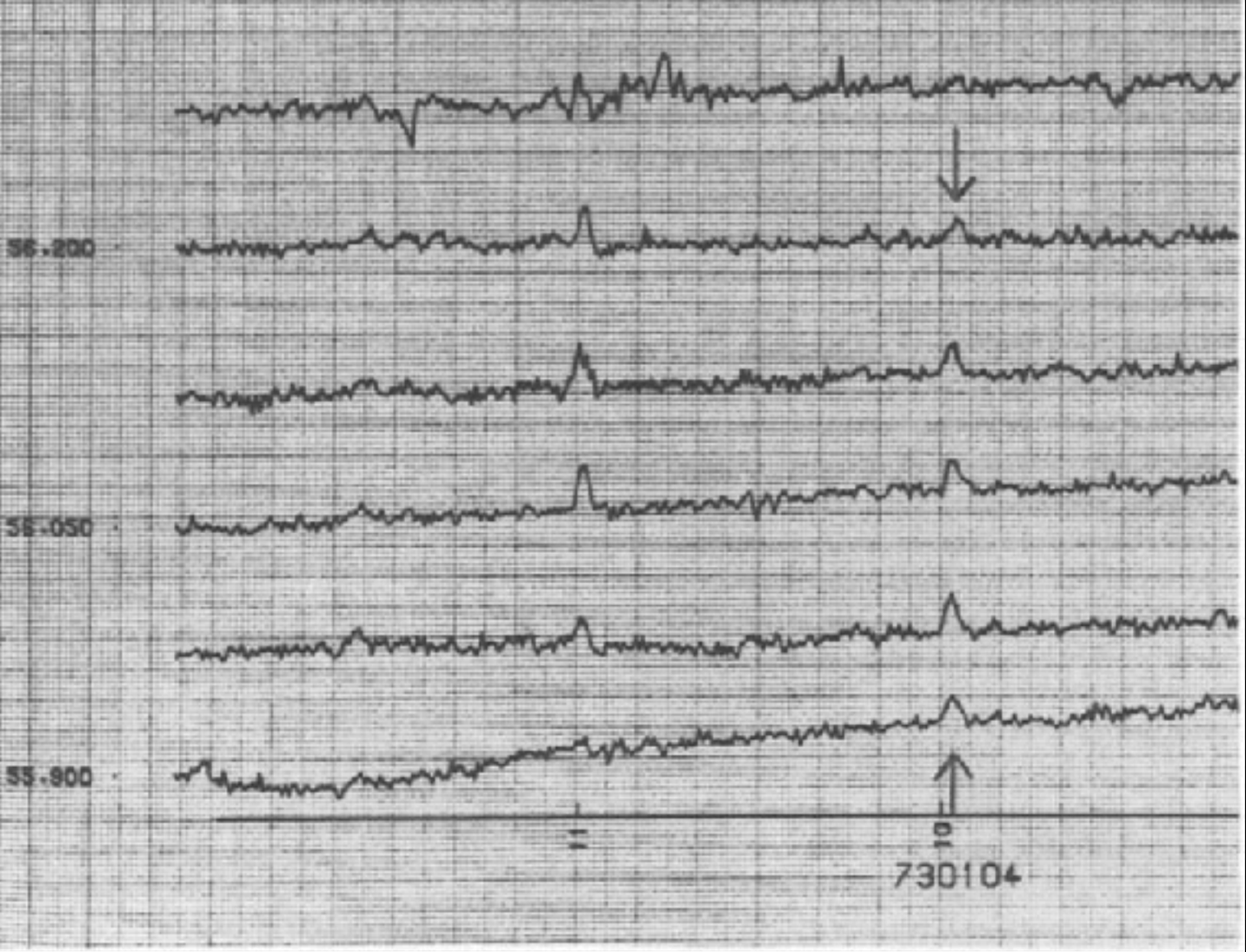
and I have magnification only for $\alpha\le\beta_E$ - that is separation lens path has a maximum

$$a_{\max}=EL\,(\alpha=\beta_E)=EL\sqrt{\frac{4\,M\,G\,LS}{ES\,EL\,c^2}}=\frac{1}{c}\sqrt{\frac{4\,M\,G\,LS\,EL}{ES}}=$$

$$=\frac{1}{c}\sqrt{\frac{4\,M\,G\,r_L(S)a(t_S)r_E(L)a(t_L)}{r_E(S)a(t_S)}\frac{a(t_E)}{a(t_E)}}=\frac{1}{c}\sqrt{\frac{4\,M\,G\,r_L(S)\,r_E(L)}{r_E(S)}a(t_E)}\frac{1}{\sqrt{1+z_L}}$$

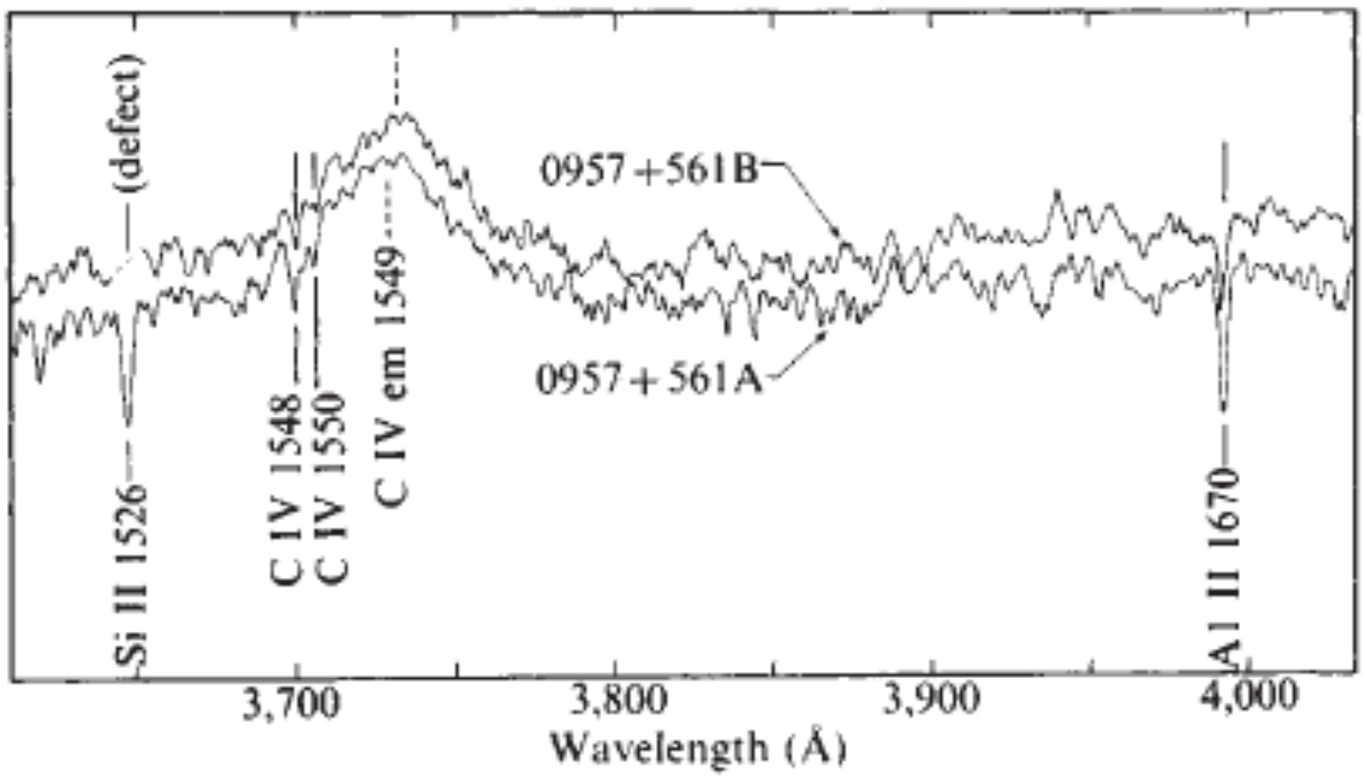


Wambsganss 1998



Walsh - 1973 Tracing

From the
0957+561:
The unpublished Story



Walsh et al. 1979

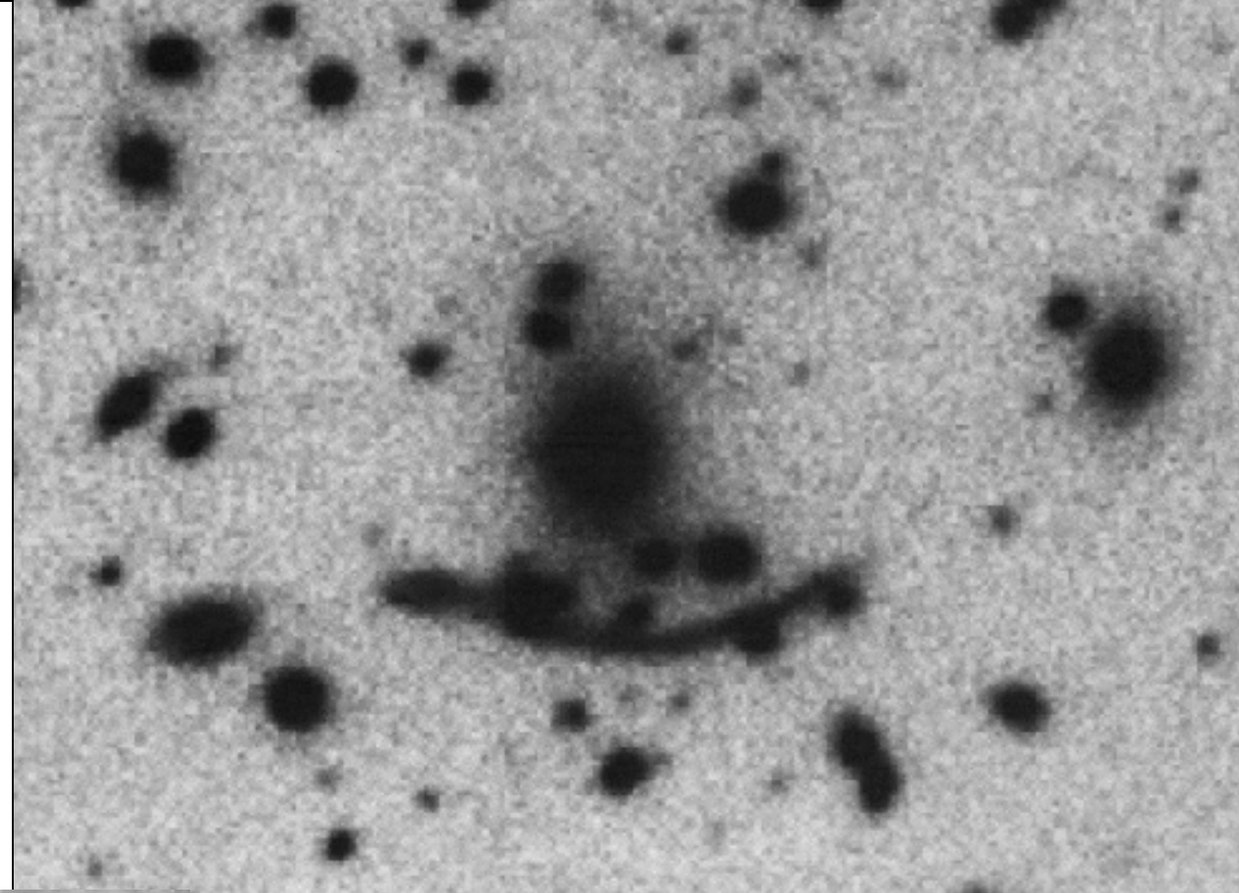
The coincidence in redshift of the absorption lines convinced the authors they were dealing with lensing

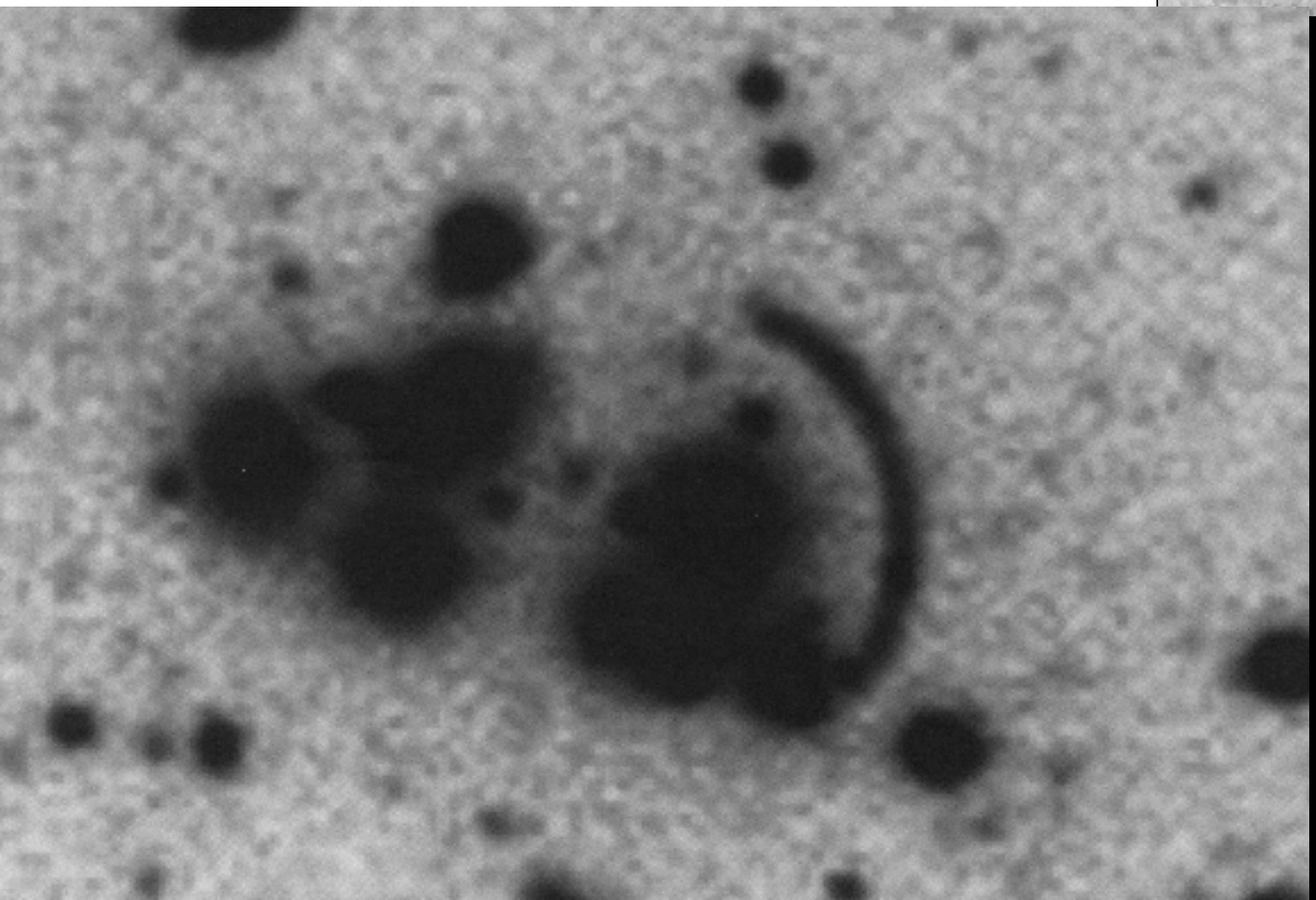
Giant Luminous Arcs in Galaxy Clusters

R. Lynds (KPNO/NOAO), V. Petrosian (Stanford U.)

1986 AAS meeting

We announce the existence of a hitherto unknown type of spatially coherent extragalactic structure having, in the two most compelling known examples, the common properties: location in clusters of galaxies, narrow arc-like shape, enormous length, and situation of center of curvature toward both a cD galaxy and the apparent center of gravity of the cluster. The arcs are in excess of 100 Kpc in length, have luminosities roughly comparable with those of giant E galaxies, and are distinctly bluer than E galaxies - especially so in one case. Interpretations of the nature of the arcs are discussed within the framework of available data.

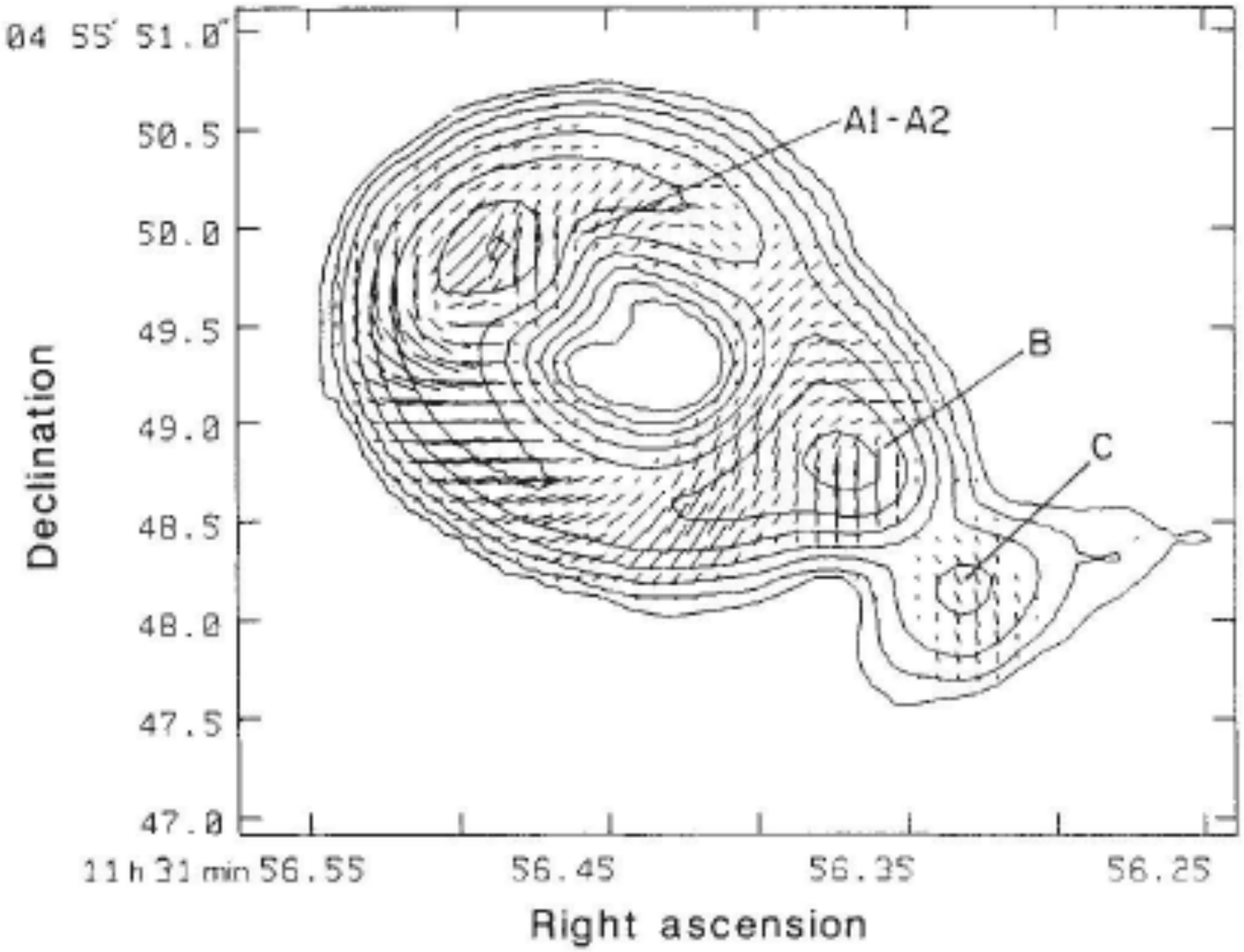




A 370

Lynds & Petrodsian
1989

Cl 2244 -02

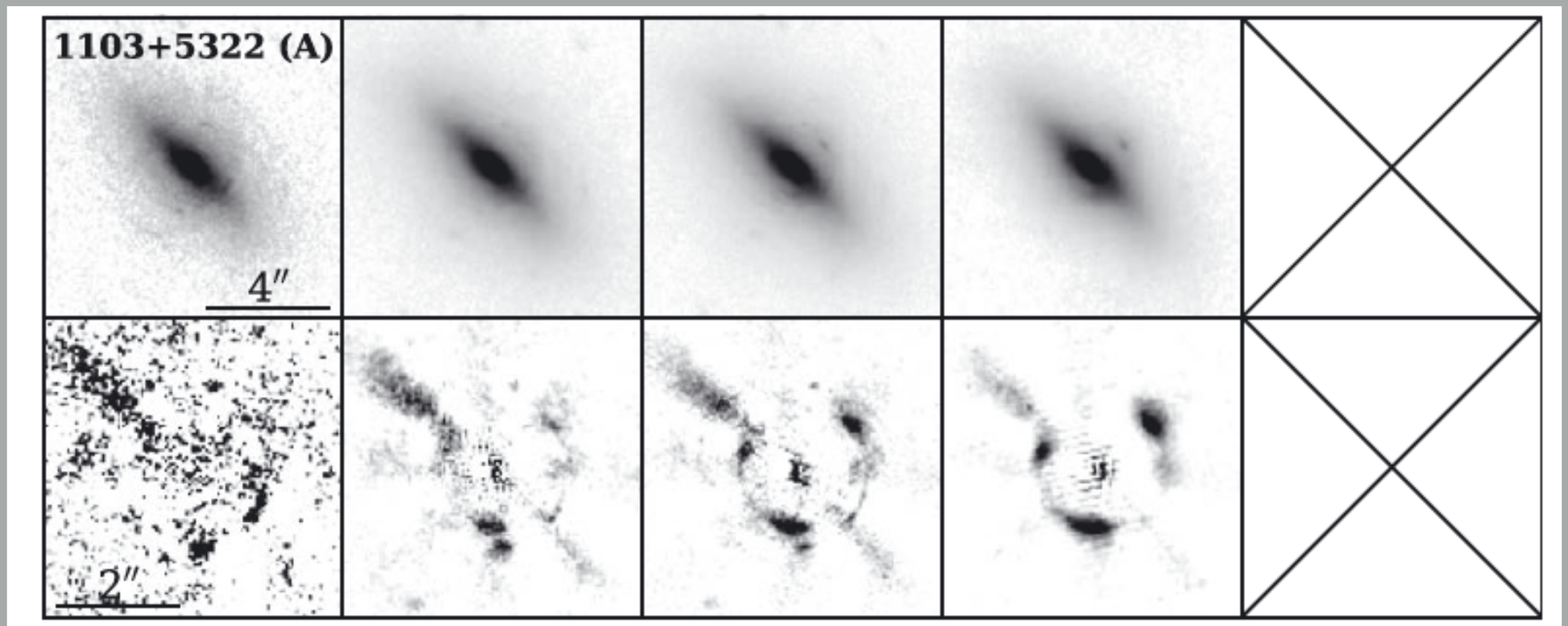


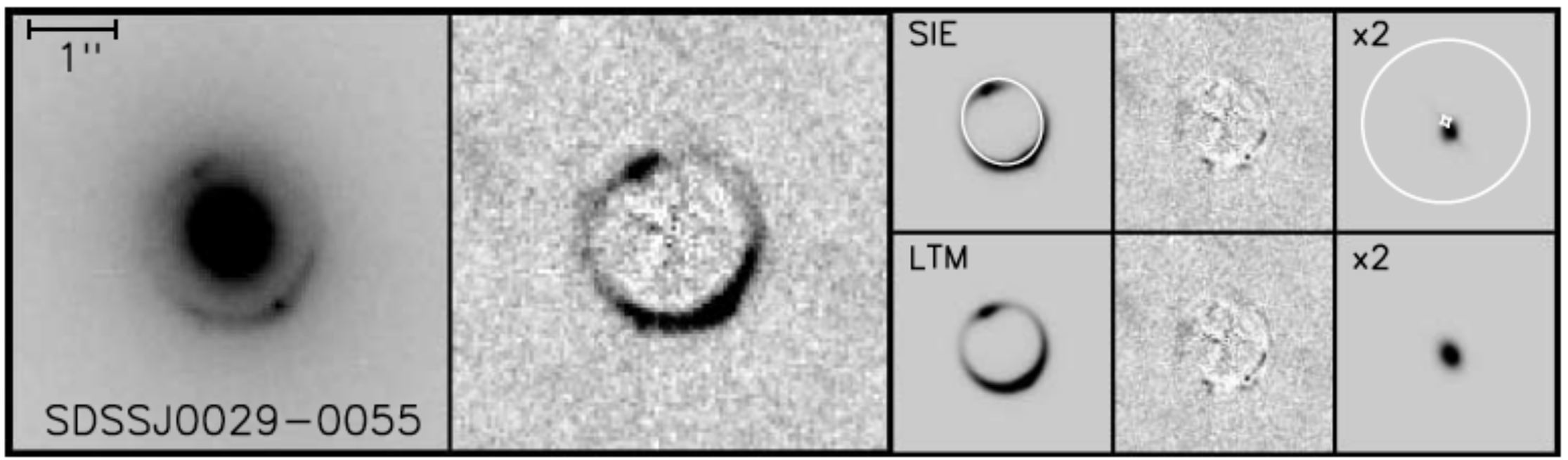
Hewitt et al. 1989

First Detection of Einstein Ring

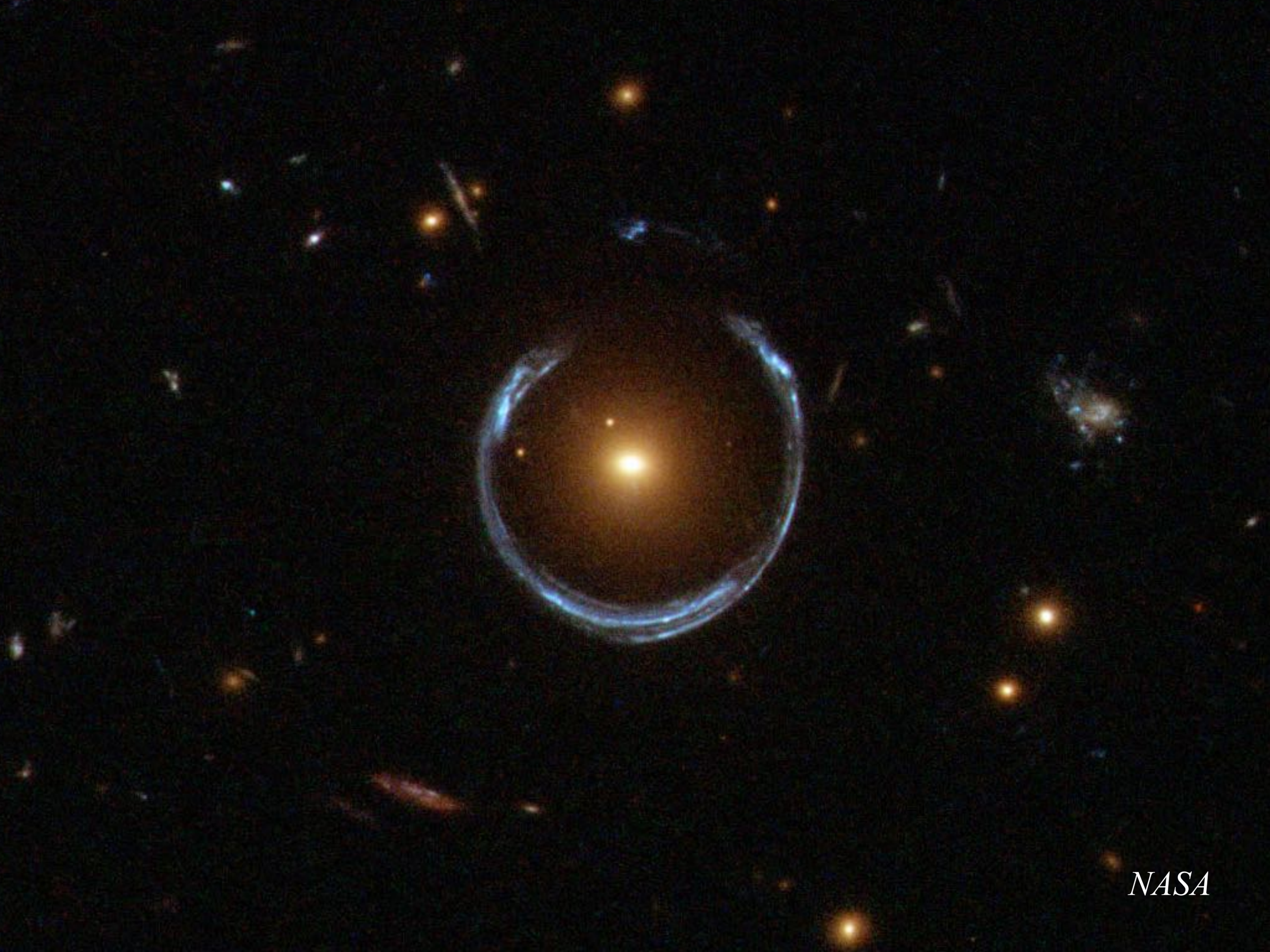
The brightness distribution is well described any an ellipse (2.2"x1.6"). In addition compact source North (A1-A2) double and South (C). The radio morphology of this source suggest that this may bean example of "Einstein Ring".

Treu et al. 2011 SWELLS survey

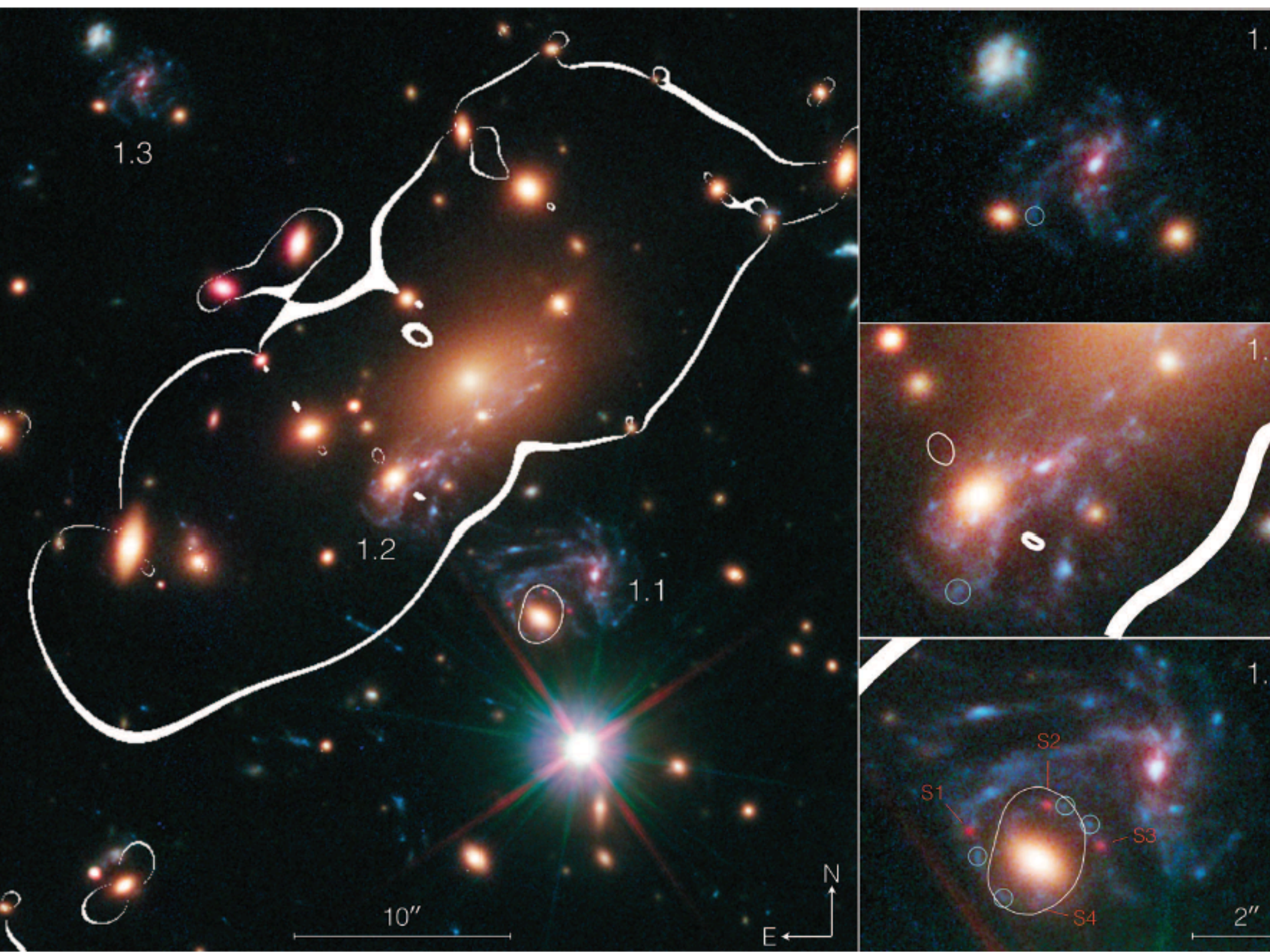


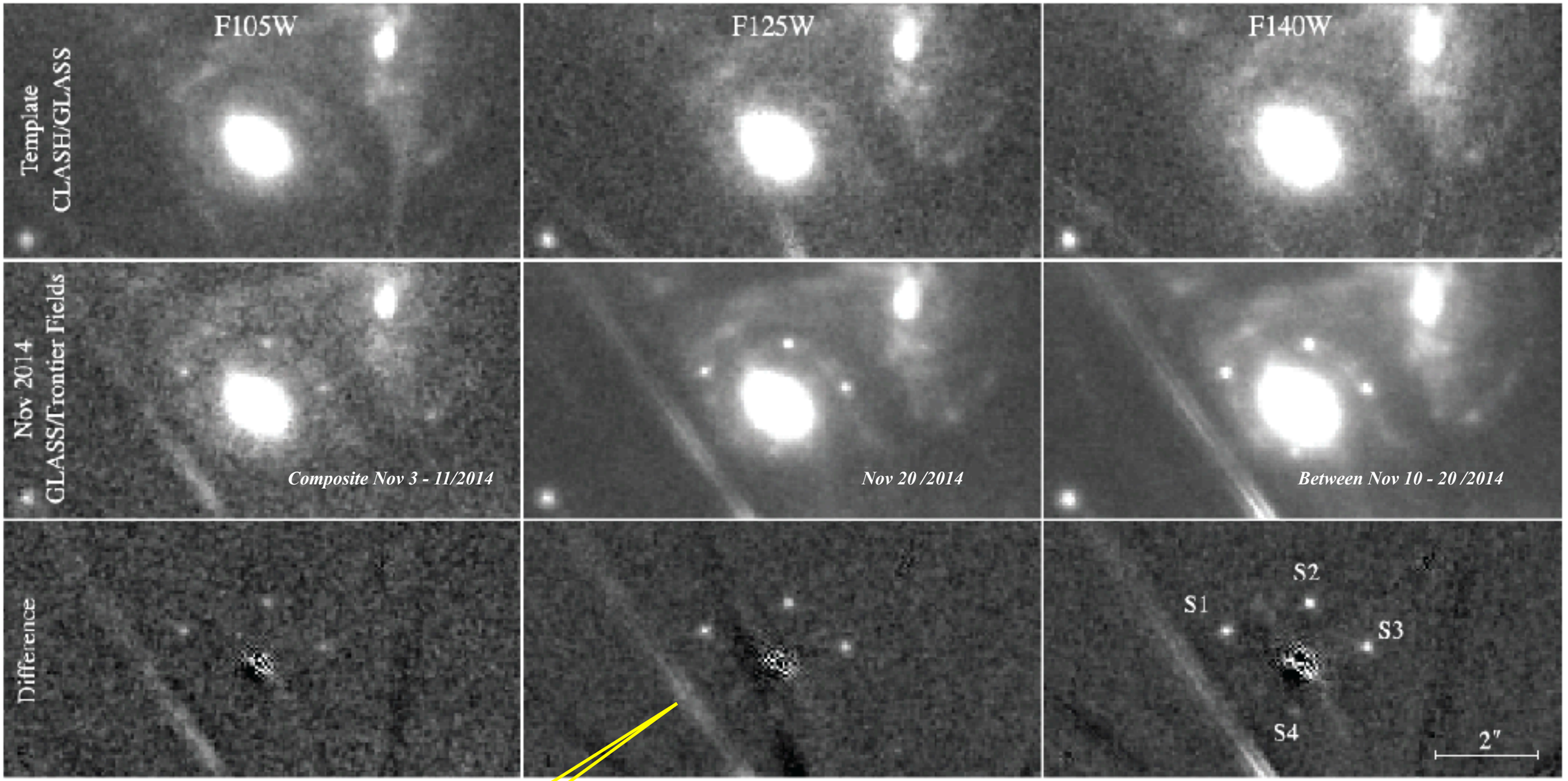


ACS survey -Bolton et al.2008



NASA


1.3
1.2
1.1
10″
N
E
S1
S2
S3
S4
2″



Kelly et al. 2015

Diffraction Spikes

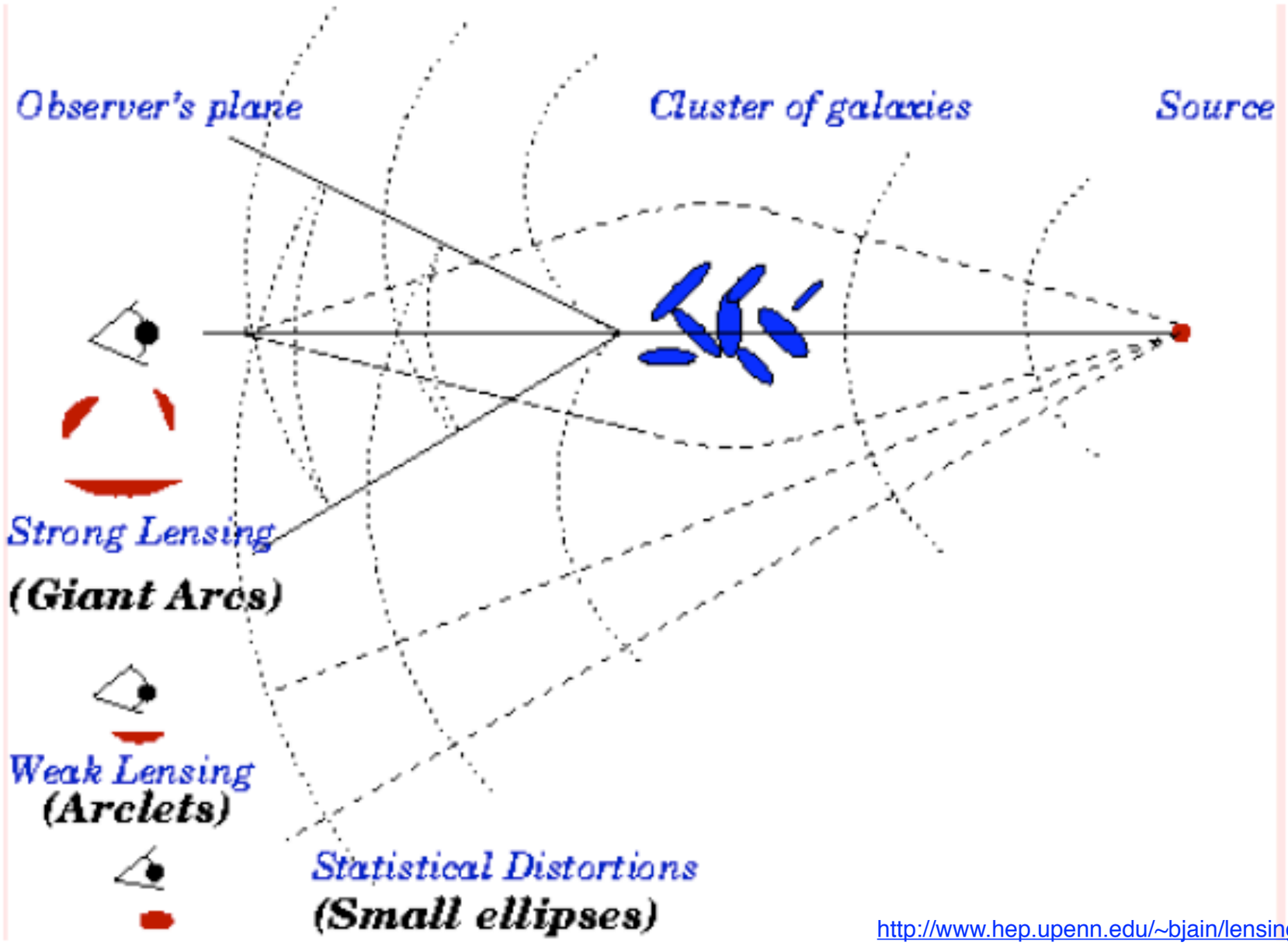
The last row is the subtraction between the second and the first row.

Cluster - Elliptical z = 0.54 // Spiral SN z= 1.49

Brief mention to analysis

&

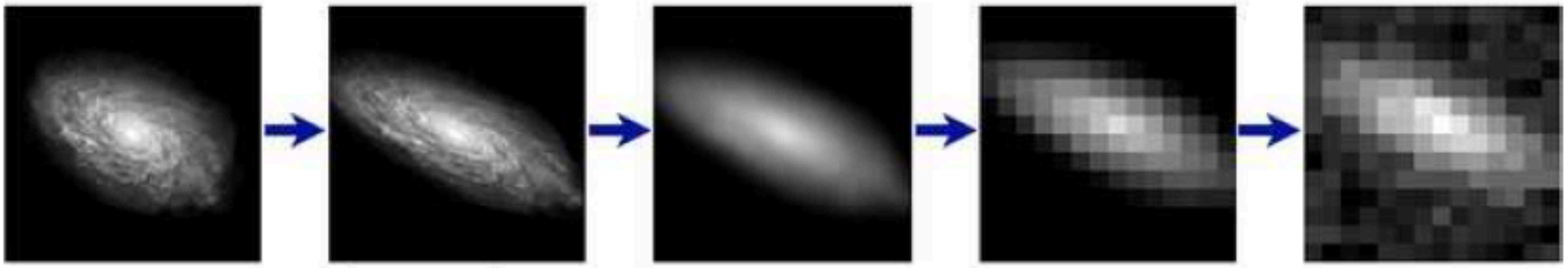
results



http://www.hep.upenn.edu/~bjain/lensing.html

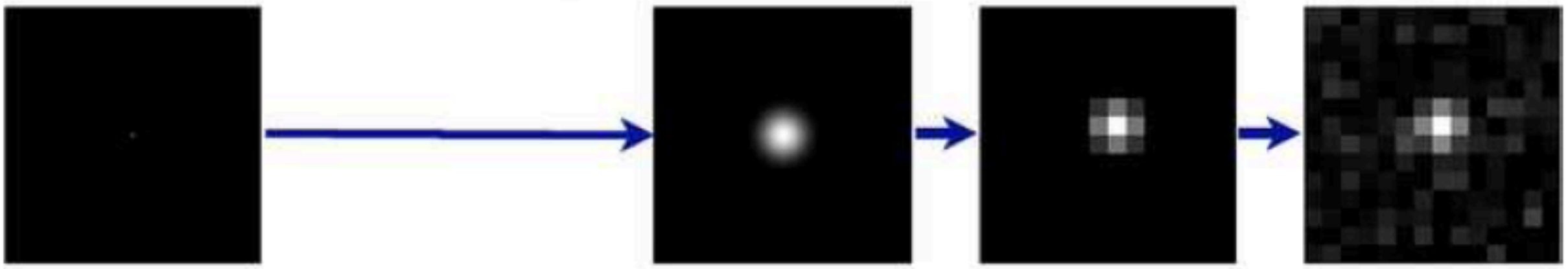
The Forward Process.

Galaxies: Intrinsic galaxy shapes to measured image:



Intrinsic galaxy (shape unknown) | Gravitational lensing causes a ***shear (g)*** | Atmosphere and telescope cause a convolution | Detectors measure a pixelated image | Image also contains noise

Stars: Point sources to star images:



Intrinsic star (point source) | Atmosphere and telescope cause a convolution | Detectors measure a pixelated image | Image also contains noise

From Brindle et al. 2009

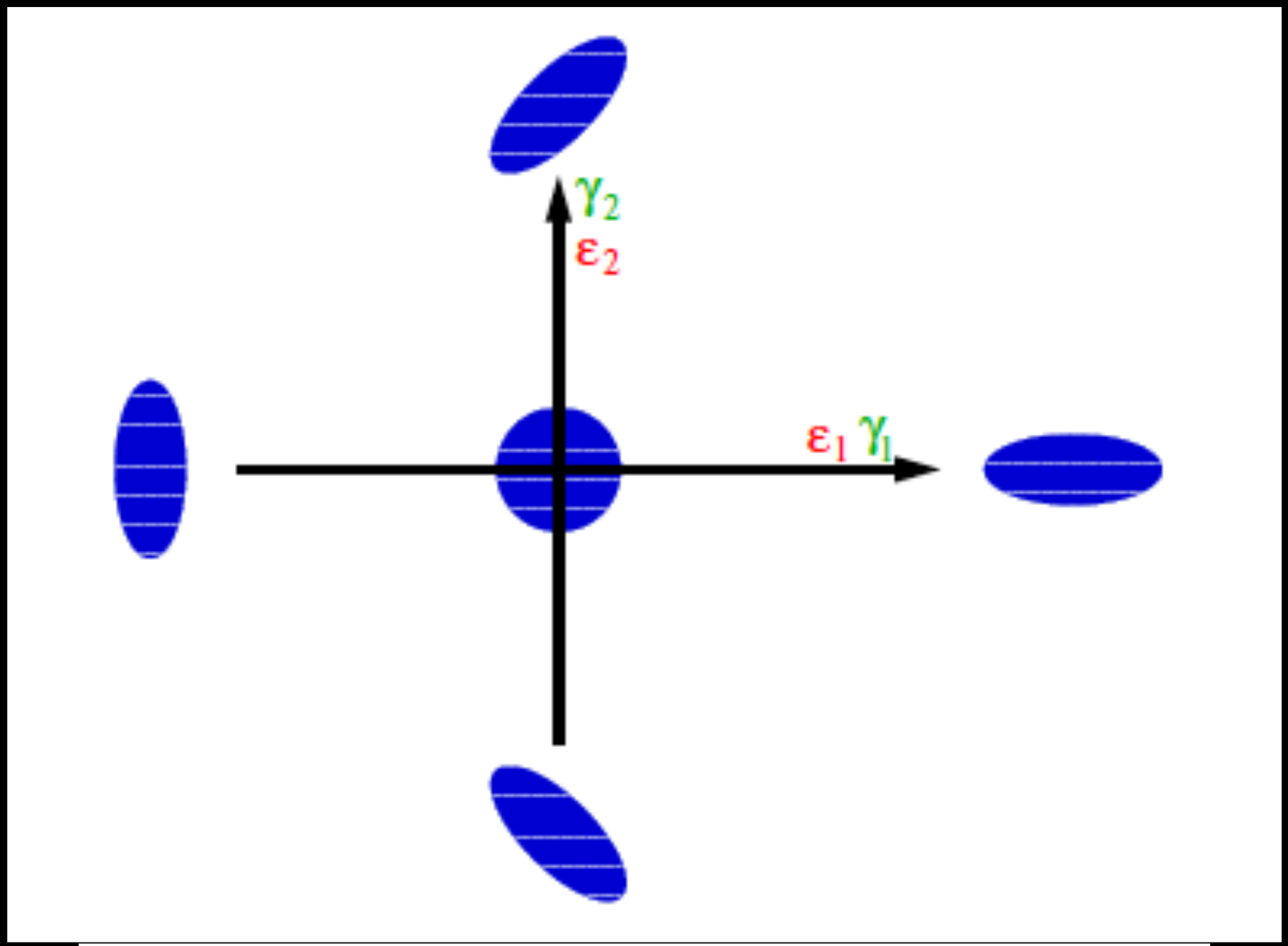
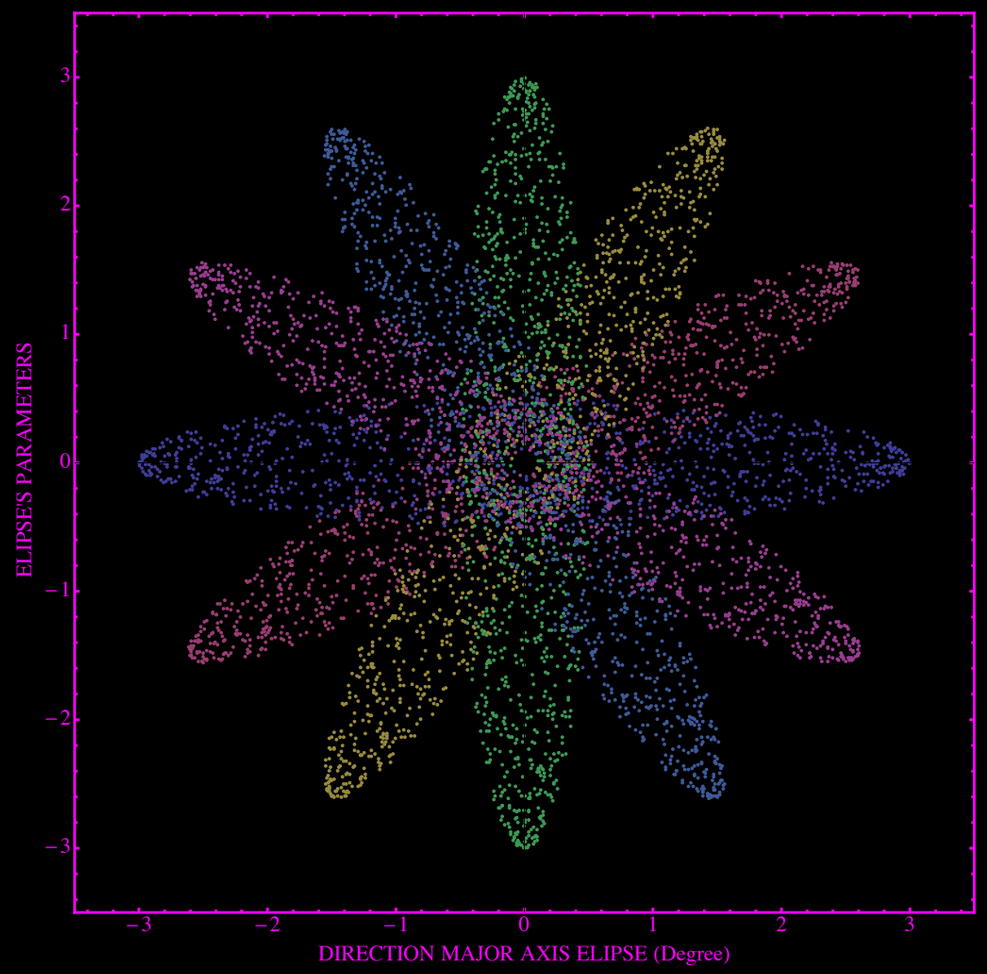


Figure 2: Illustration of the geometrical meaning of the shear γ_i and of the ellipticity ϵ_i. A positive (negative) shear component γ_1 corresponds to an elongation (compression) along the x-axis. A positive (negative) value of the shear component γ_2 corresponds to an elongation (compression) along the $x = y$ axis. The ellipticity of an object is defined to vanish if the object is circular (*center*). The ellipticity components ϵ_1 and ϵ_2 correspond to compression and elongations similar to those for the shear components.



very simply to have an idea

$$Q_{11} = \frac{\sum_x \sum_y I(x,y)(\bar{x}-x)(\bar{x}-x)}{\sum I(x,y)}$$

$$Q_{12} = \frac{\sum_x \sum_y I(x,y)(\bar{x}-x)(\bar{y}-y)}{\sum I(x,y)}$$

$$Q_{22} = \frac{\sum_x \sum_y I(x,y)(\bar{y}-y)(\bar{y}-y)}{\sum I(x,y)}$$

$$e_1 = \frac{Q_{11}-Q_{22}}{Q_{11}+Q_{22}}$$

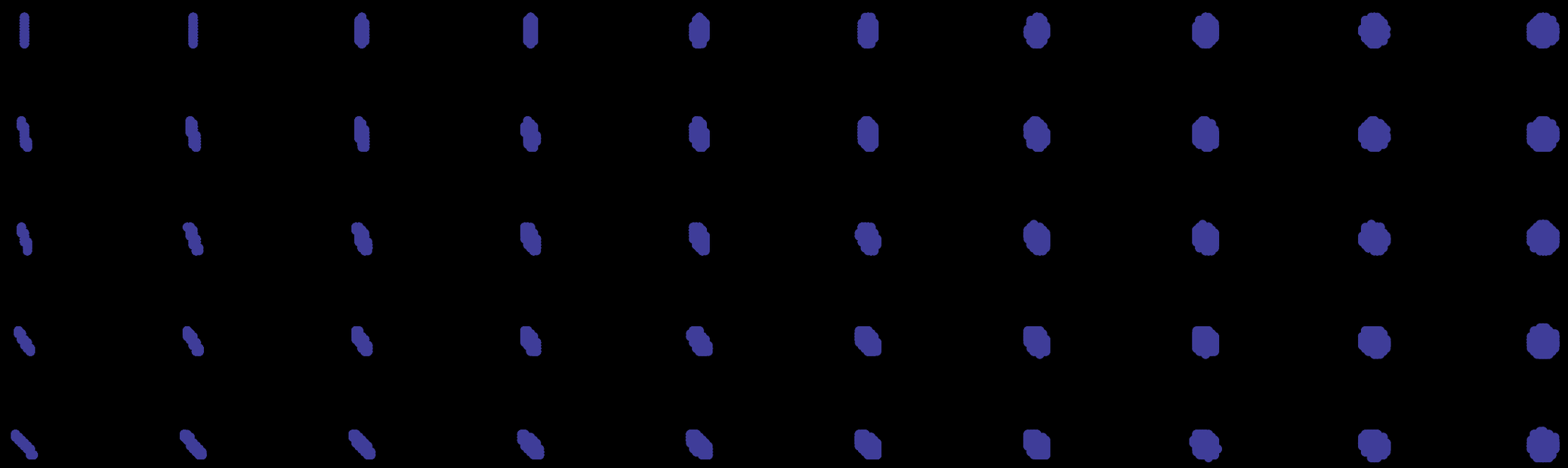
$$e_2 = \frac{2Q_{1,2}}{Q_{11}+Q_{22}}$$

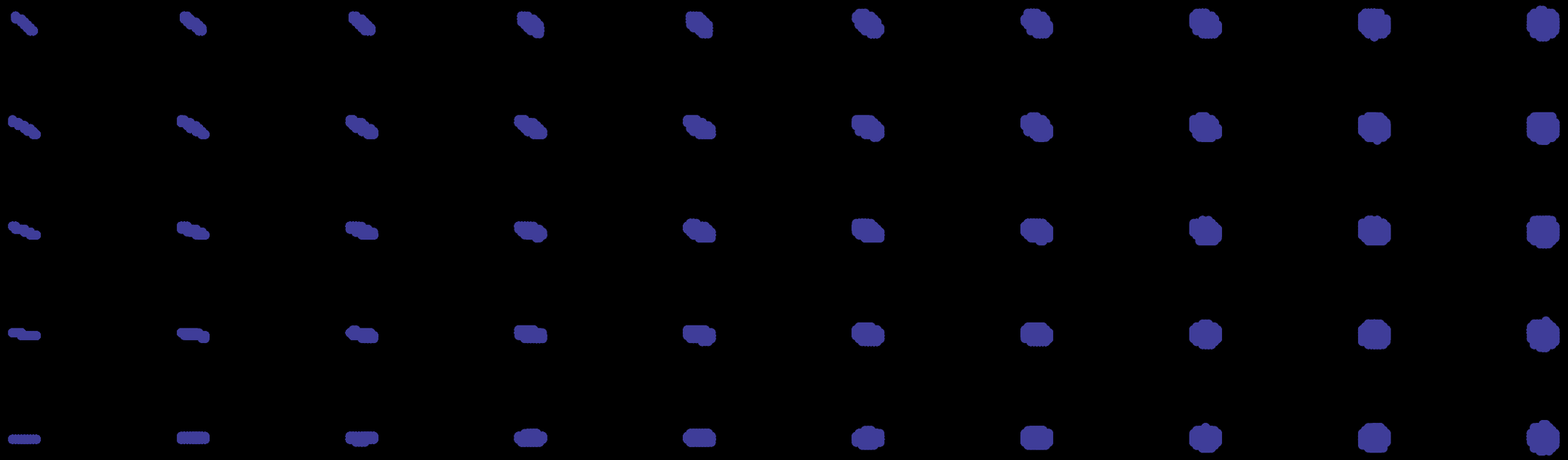
$$\left.\begin{aligned} e_1 &= \frac{a-b}{a+b}\cos 2\theta \\ e_2 &= \frac{a-b}{a+b}\sin 2\theta \end{aligned}\right\} \quad \theta = \frac{1}{2}\arctan\frac{e_2}{e_1}$$

Position angle

$e_1 = -1$

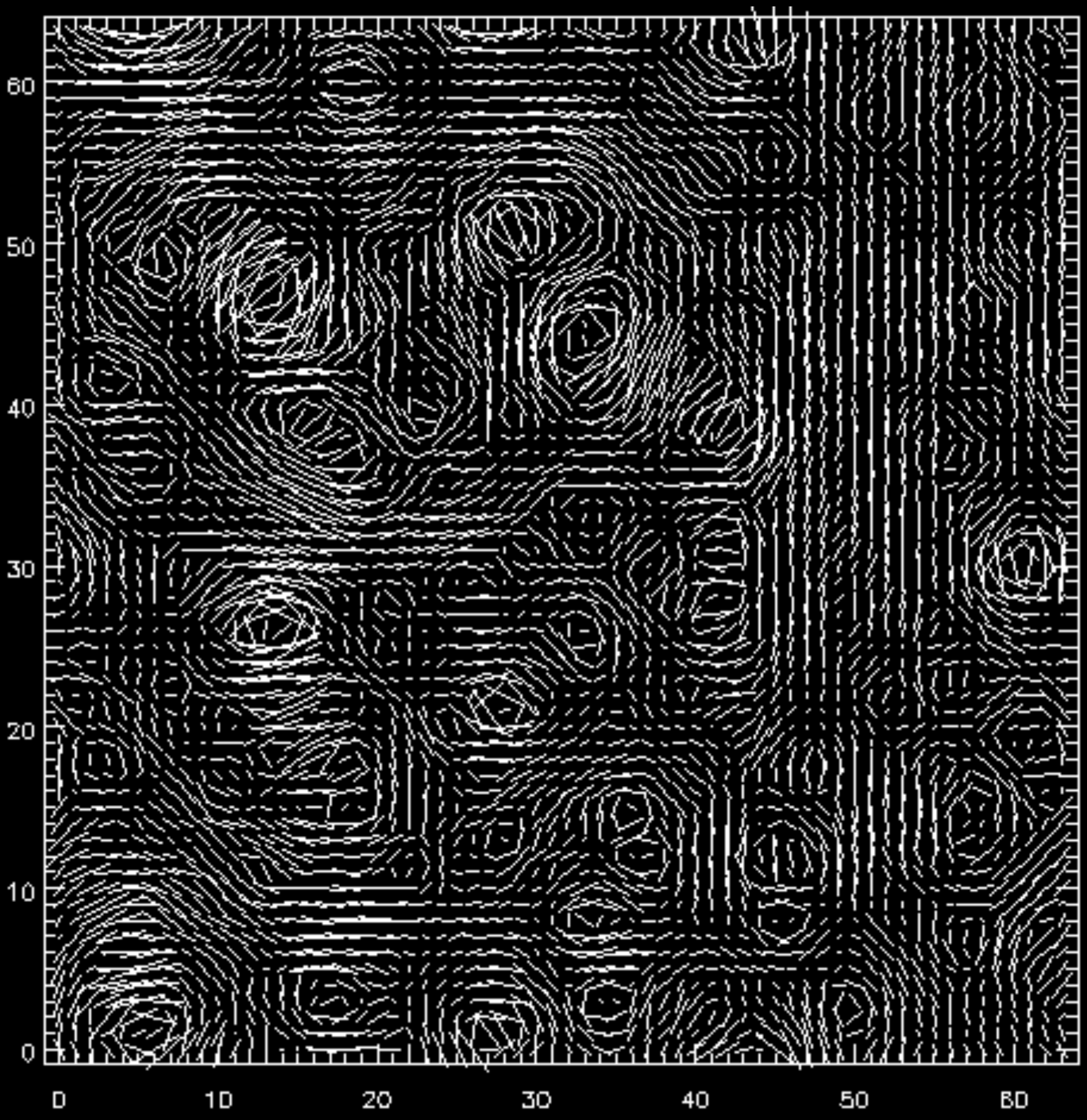
$e_2 = 0$



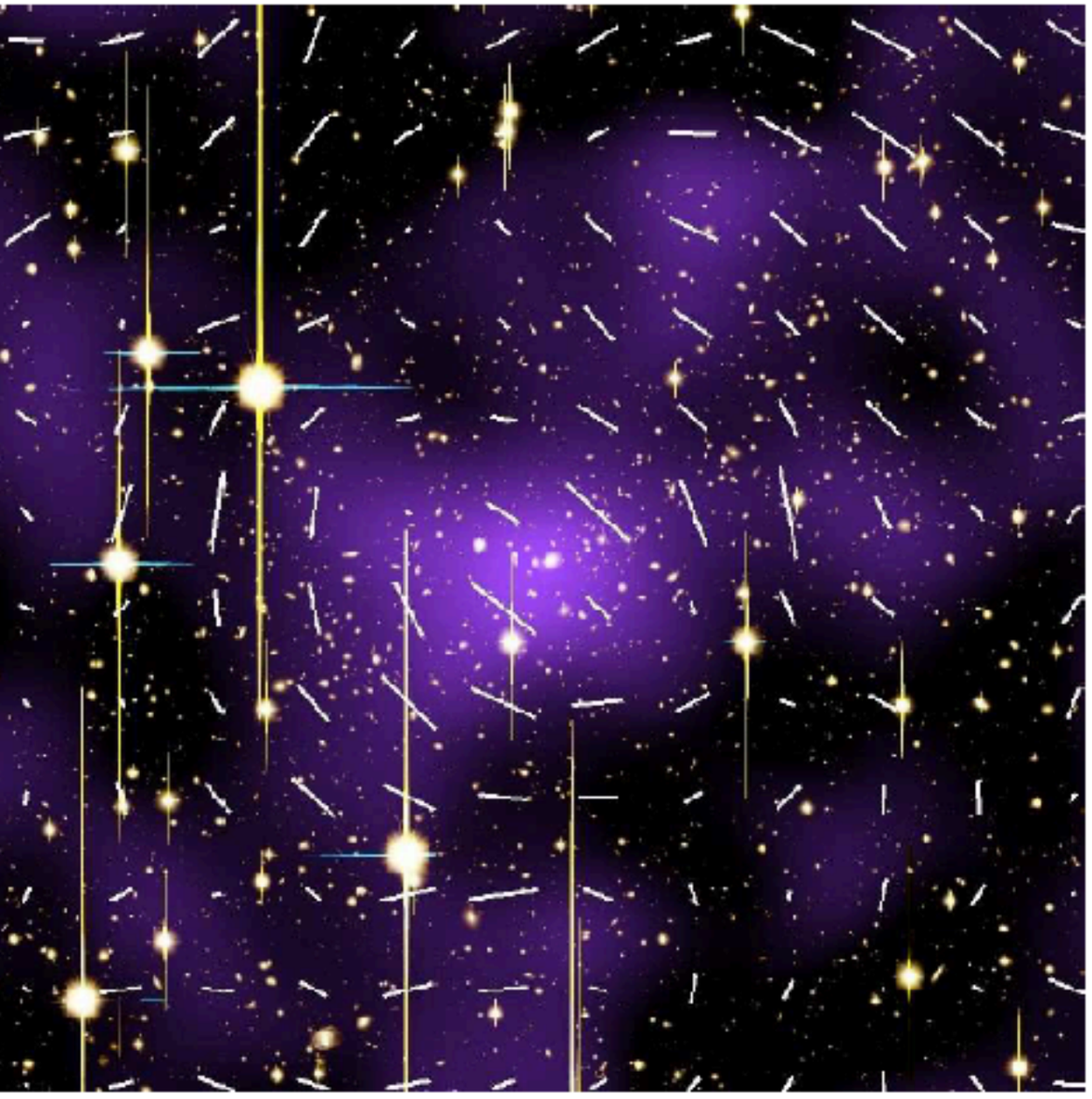


$e_1 = 1$

$e_2 = 0$



Jain 1999



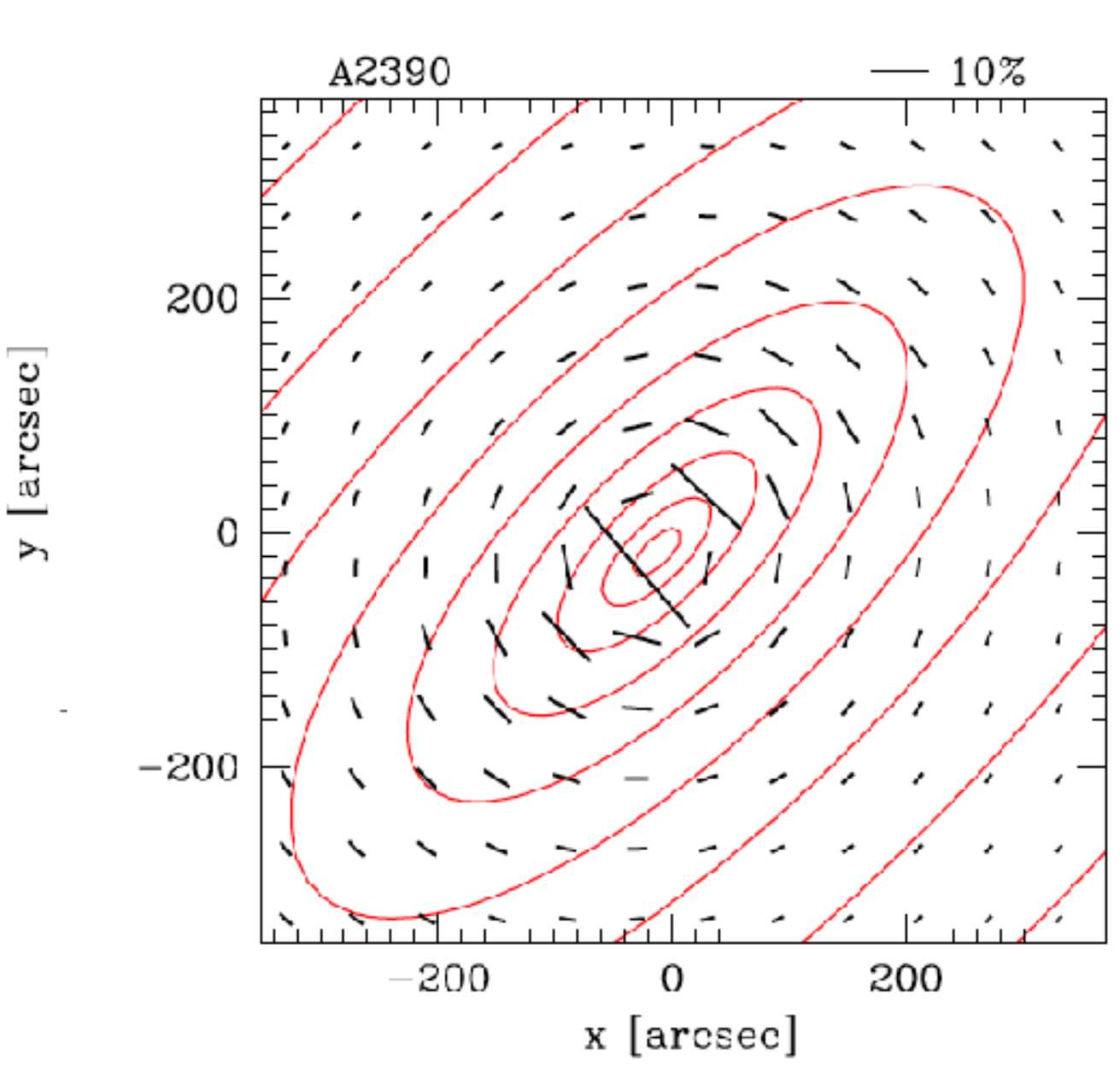


Figure 1. *Left panel:* An example of our weak lensing measurement for A2390. The size of Each panel is $12' \times 12'$. The stick in each $1' \times 1'$ pixel shows the distortion field estimated from background galaxy images contained within the pixel, where a background galaxy image is deformed along the stick direction, and the length is proportional to the shear amplitude. The shear field in this panel is smoothed with a Gaussian with the full width at half maximum of $\simeq 1.6'$ for illustrative purpose. Overplotted is the surface mass density map reconstructed from the weak lensing shear measurement (see Okabe et al. 2010). North is up and East is left. *Right panel:* The shear field predicted by our best-fit elliptical NFW model (see also Figure 2 and Table 1), while the contours are the isodensity map. The best-fit ellipticity of the projected mass density is $e \equiv 1 - b/a = 0.598$.

Oguri et al. 2010

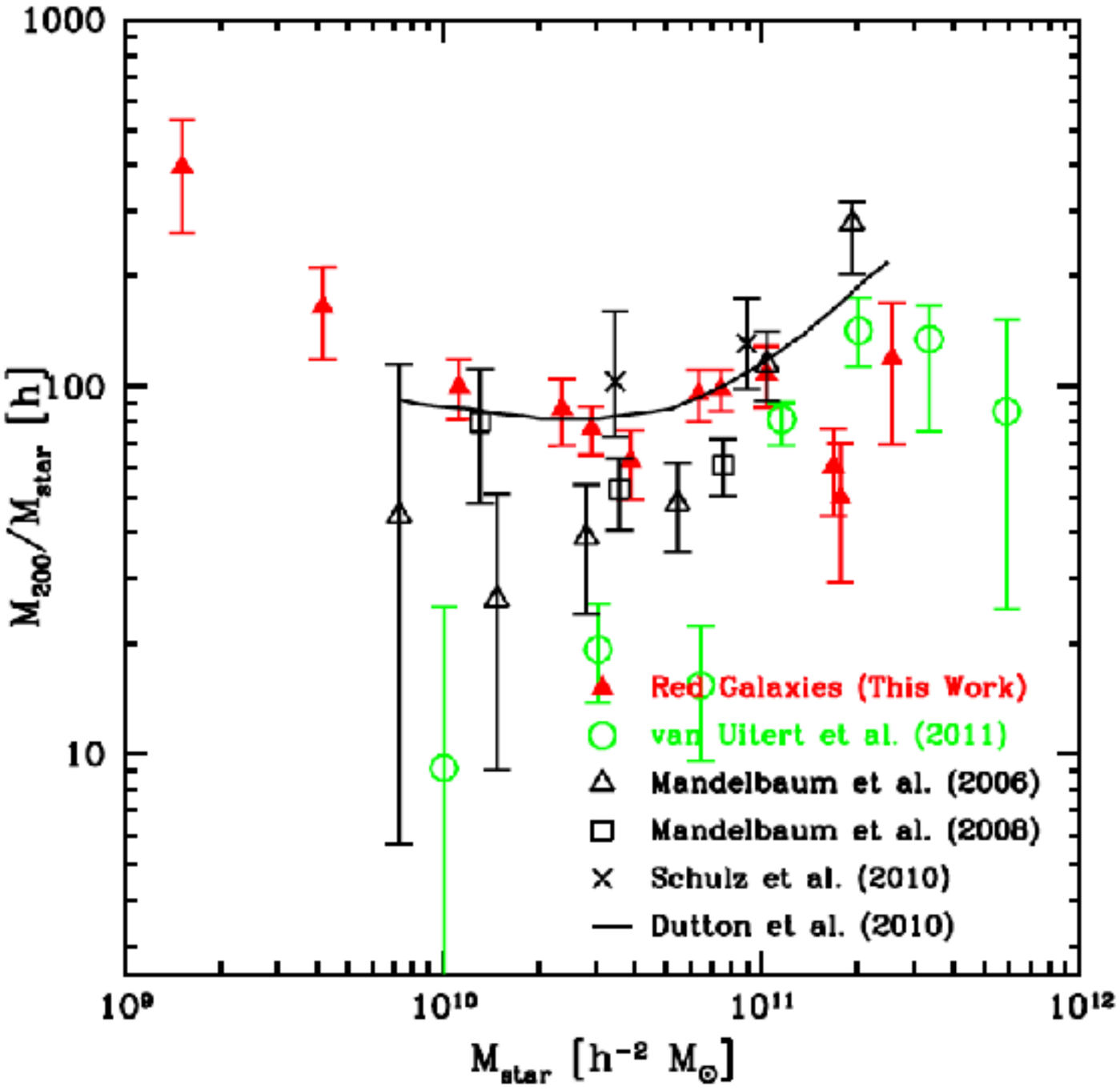


Figure 33. Stellar Mass vs. $M_{200}/M_{\rm star}$ ratio for red galaxies converted to $z = 0$. The red triangles denote our red galaxies. We have added the results of Mandelbaum et al. (2006c) (open triangles), Mandelbaum, Seljak & Hirata (2008) (open squares), Schulz, Mandelbaum & Padmanabhan (2010) (black crosses) and Dutton et al. (2010) (black solid line), see also fig. 1 in Dutton et al. (2010). We further include the results of van Uitert et al. (2011) from their fig. 14 as green open circles.

Canada-France-Hawaii Telescope
Legacy Survey Wide

From a 12 million object catalogue
$0.05 < z_s < 2$
$0.05 < z_d < 1$

Tested profiles:
Iso, Iso truncated, NFW

$$M_{Tot} = 2.32^{+0.28}_{-0.25}\, 10^{12} h^{-1} M_\odot$$

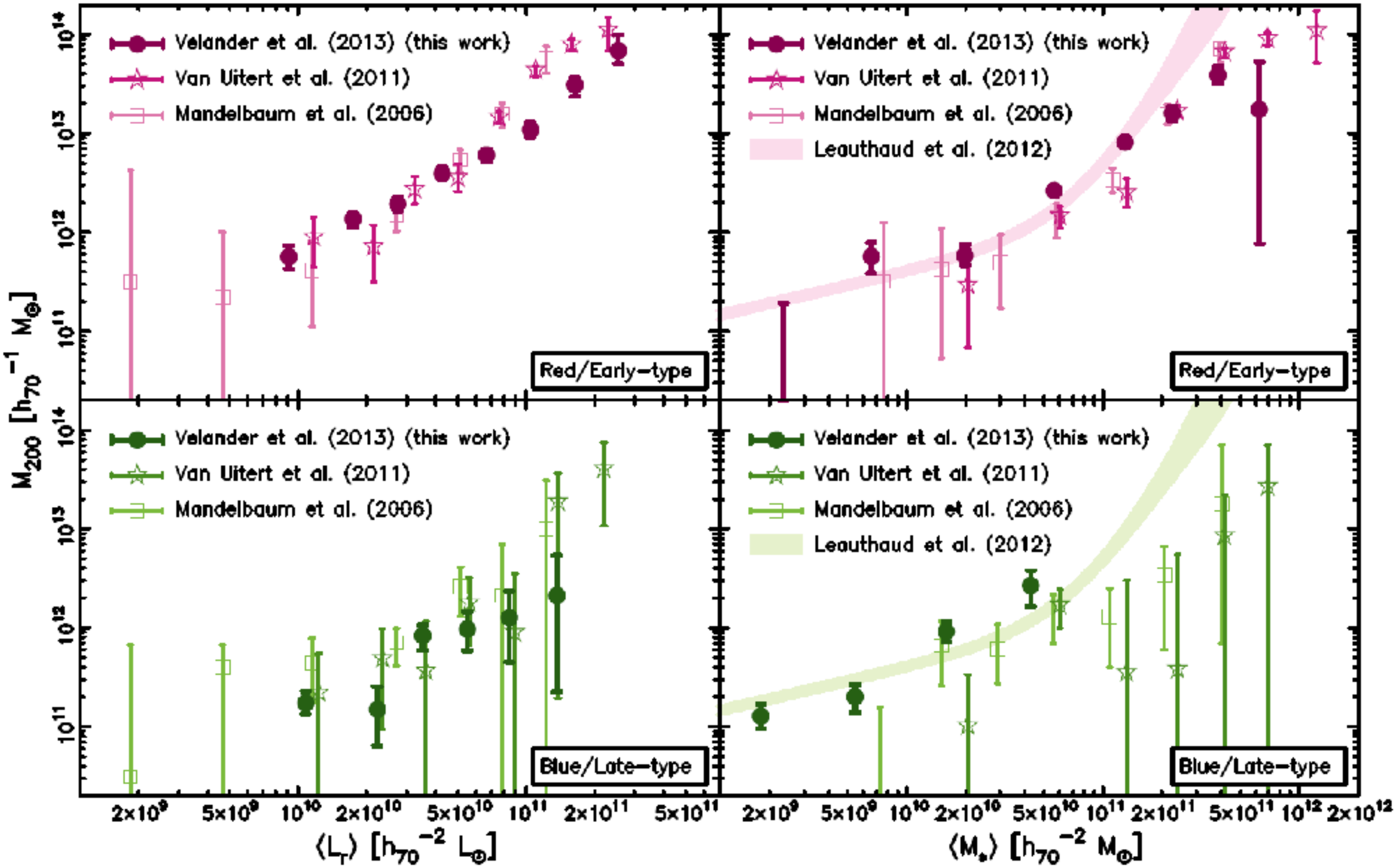
$$M/L = 178^{+22}_{-19} h\, M_\odot/L_\odot$$

$$at\ L = 1.6\, 10^{10} h^{-2} L_\odot$$

Transformation used

$$Log_{10}(M_{star}) = 1.093\, Log_{10} L - 0.573$$

Brimioulle et al. 2013



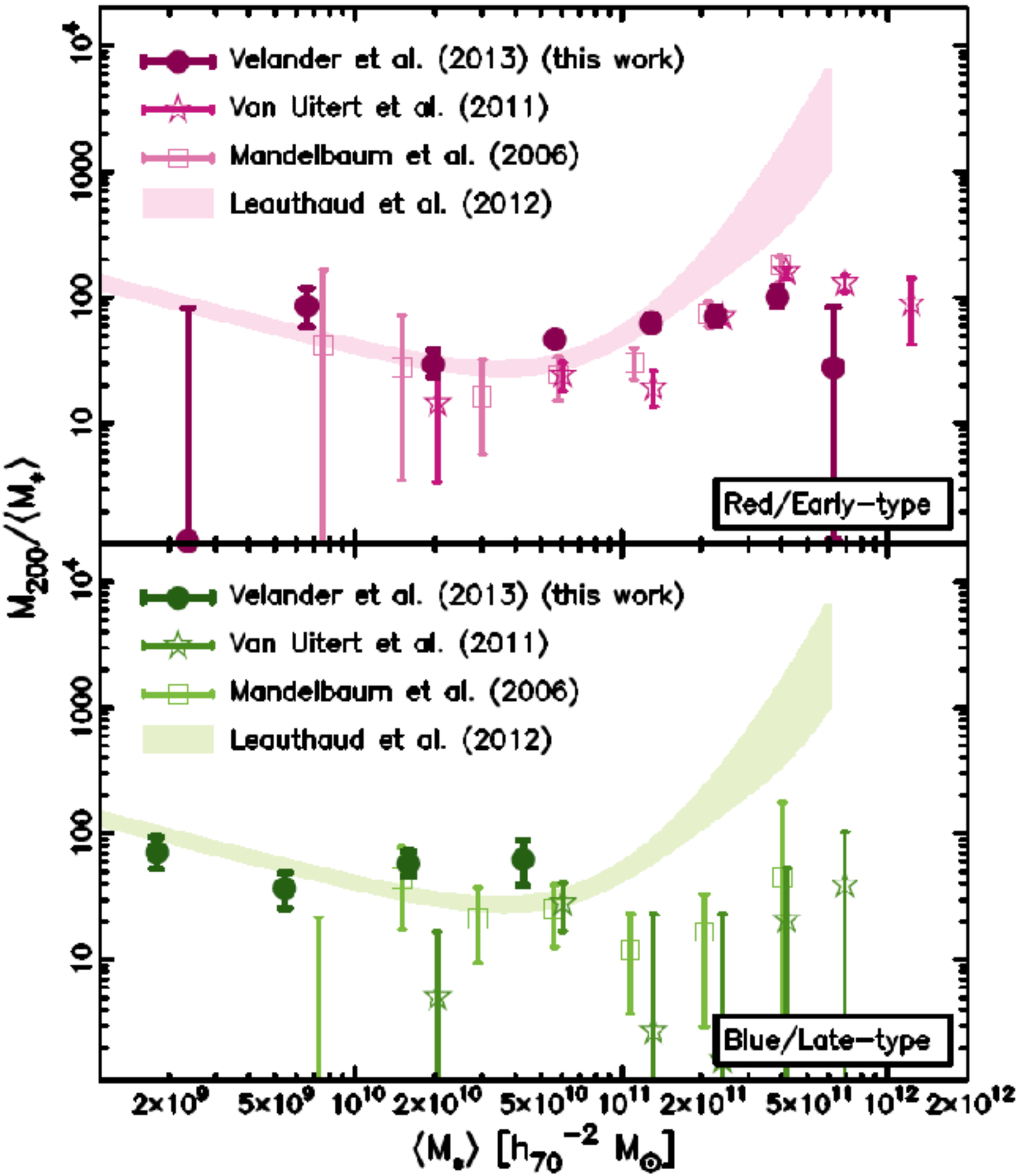
Velander et al.2013

$$0.2 < z_{lens} < 0.4$$

$$L_{Fid} = 10^{11} h_{70}^{-2}\ L_{\odot} \begin{cases} Red\ Gal \begin{cases} slope\ 1.32 \pm 0.06 \\ M_{200}\ 1.19^{+0.06}_{-0.07}\ 10^{13} h_{70}^{-1}\ M_{\odot} \end{cases} \\ Blue \begin{cases} slope\ 1.09^{+0.20}_{-0.13} \\ M_{200} = 0.18^{+0.04}_{-0.05}\ 10^{13} h_{70}^{-1}\ M_{\odot} \end{cases} \end{cases}$$

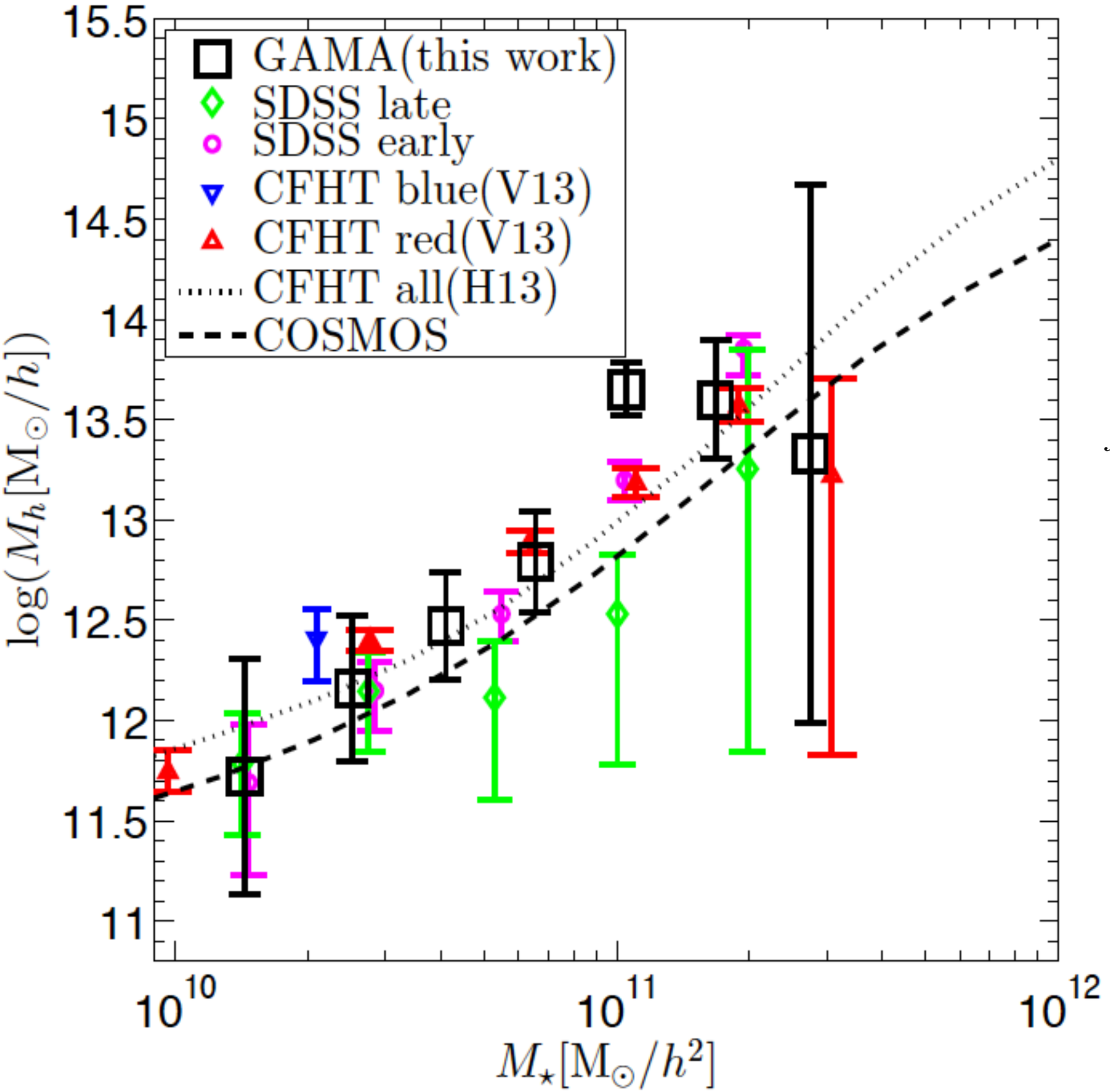
$$M_{Fid} = 210^{11} h_{70}^{-2}\ M_{\odot} \begin{cases} Red\ Gal \begin{cases} slope\ 1.36^{+0.06}_{-0.07} \\ M_{200} = 1.43^{+0.11}_{-0.08}\ 10^{13} h_{70}^{-1}\ M_{\odot} \end{cases} \\ Blue \begin{cases} slope\ 0.98^{+0.08}_{-0.07} \\ M_{200} = 0.84^{+0.20}_{-0.16}\ 10^{13} h_{70}^{-1}\ M_{\odot} \end{cases} \end{cases}$$

Velander et al.2013



Efficiency or
where are the baryons?

$$\eta = \frac{M_*}{M_{DM}} \frac{\Omega_M}{\Omega_b} \approx \begin{cases} \eta_{\mathrm{Red}} = 9\% \\ \eta_{Blue} = 15\% \end{cases}$$



Han 2014

The central halo mass - stellar mass relation measured from weak lensing

Baryon efficiency at $M* \sim 10^{11} M_\odot$

$\boldsymbol{\eta}$ *~10%*

(or missing baryons?)

Conclusions

- *DM Halos (M_{200}) are rather large ~ 100 - 200 kpc*
- *DM Halos have masses of the order of 10^{13} $M_{\odot}$*
- *The stellar mass correlates with the DM Halo mass*
- *M/L ~ 20 to 140 (Blue to red) - Based Ver - Brim*
- *The efficiency in Baryons production is of ~ 10% - 15%*

Conclusions

- *Best stellar mass tracer ?*
- *Red and Blue galaxies selection ? Morphology, color ….*
- *Best modeling of the Halo mass ?*
- *Baryons location ?*

Non tutti quelli che vagano si sono persi

NOT ALL THOSE WHO WANDER ARE LOST

[J.R.R. Tolkien]

APPENDIX